Swaps

Use a swap to transform an **asset**

Suppose MS owns $100 million in bonds that will provide interest at 4.7% per annum over the next three years. Afterwards MS enters into the swap and has the following **three** sets of cash flows

- It receives 4.7% on the bonds
- It receives LIBOR under the terms of the swap (from some other party, let us call it the third party)
- It pays 5% under the terms of the swap (to some other party, let us call it the third party)

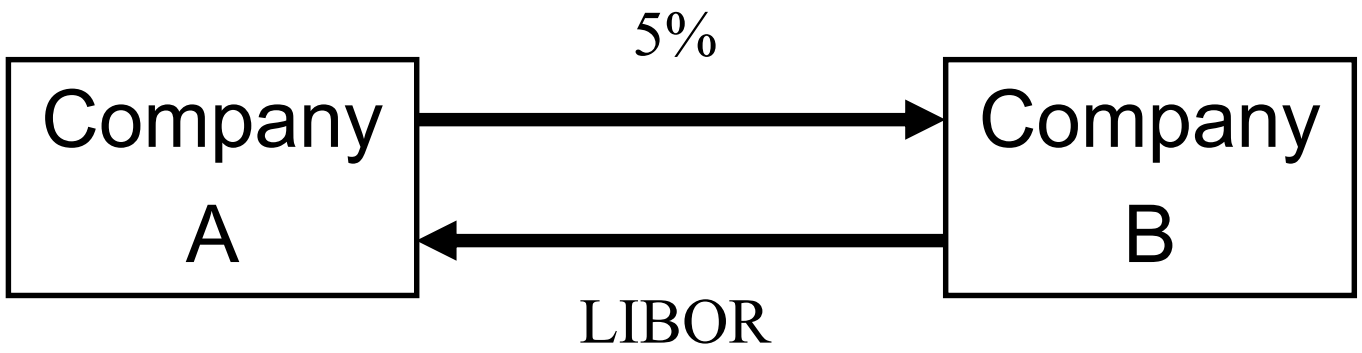
Swaps

Use a swap to transform an **asset**

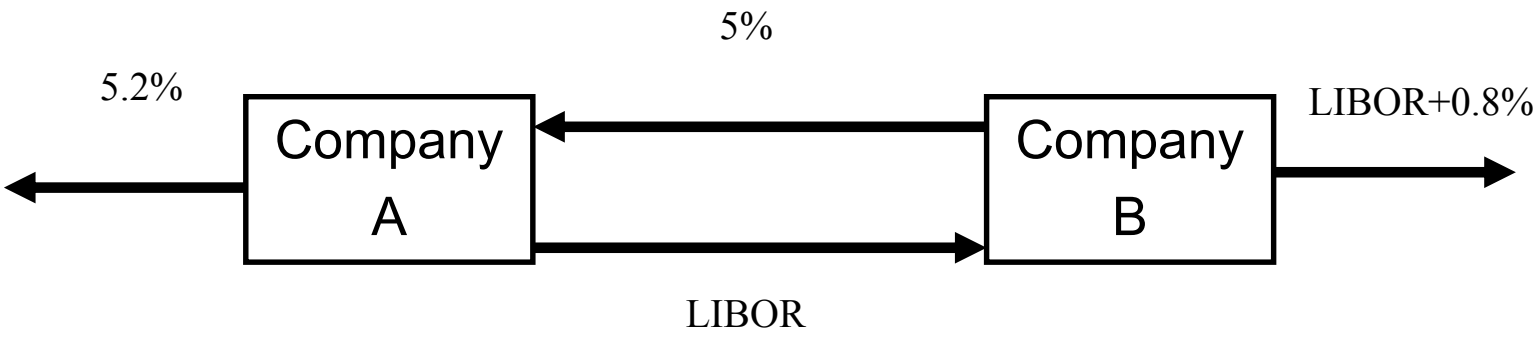
Suppose Intel has an investment of $100 million that yields LIBOR minus 20 basis points. Afterwards Intel enters into the swap and has the following **three** sets of cash flows

- It receives LIBOR minus 20 basis points on its investment
- It pays LIBOR under the terms of the swap (to some other party, let us call it the third party)
- It receives 5% under the terms of the swap (from some other party, let us call it the third party)

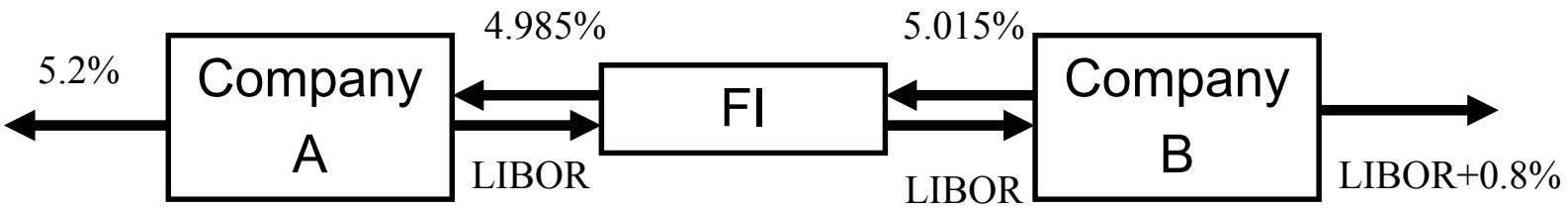
Swaps



Swap to transform Liability/Asset



Swap to transform Liability/Asset



Swaps

Usually the two entities do not come into touch with each other directly in order to arrange for the swap. They deal between themselves through a financial intermediary, such as a bank or other financial institutions

Swaps (Interest rate)

The concept of comparative advantage

For the case when a company wants to change its asset into a liability or vice versa it is undertaking that with the notion that it has some comparative advantage, either in the floating interest rate or in the fixed interest rate market. This concept of comparative advantage is true for the case when we consider an **interest swap only**

Swaps (Currency)

The concept of comparative advantage

For the case when a company wants to change its asset into a liability or vice versa it is undertaken that with the notion that it has some comparative advantage, either in the domestic currency or in the foreign currency. This concept of comparative advantage is true for the case when we consider a **currency swap only**

Swaps (Interest rate and Currency)

The concept of comparative advantage

For the case when a company wants to change its asset into a liability or vice versa it is undertaken that with the notion that it has some comparative advantage, in any one of the following

- **Floating** interest rate and/or **domestic** currency
- **Floating** interest rate and/or **foreign** currency
- **Fixed** interest rate and/or **domestic** currency
- **Fixed** interest rate and/or **foreign** currency

Swaps
(Comparative Advantage)

	Fixed	Floating
Company A	4.0%	LIBOR+0.3%
Company B	5.2%	LIBOR+1.0%

Swaps
(Comparative Advantage)

Note

- Both companies wish to borrow $100 million for 5 years and have been offered the rates as shown above. We assume that **B wants (has) to borrow at a fixed rate**, while **A wants (has) to borrow at a floating rate**
- See that the difference between the two **fixed interest rates** is **greater** than the corresponding difference between the **floating interest rates**.

Swaps
(Comparative Advantage)

Note

- B pays 1.2% more than A in the fixed rate market while 0.7% more than A in the floating rate market.
- This appears to imply that **B has a comparative advantage in the floating rate market** while **A has a comparative advantage in the fixed rate market**
- This apparent anomaly can lead to a swap agreement being negotiated between the them

Swaps
(Comparative Advantage)

Now the swap is like given below, even though practically that is not the case

- Assume A and B get in touch with each other directly
- A agrees to pay 4.0% to the outside lender
- It receives 3.95% (say) from B
- It pays LIBOR (say) to B

Swaps
(Comparative Advantage)

Now the swap is like given below, even though practically that is not the case

- Assume A and B get in touch with each other directly
- B agrees to pay LIBOR+1.0% to the outside lender
- It receives LIBOR (say) from A
- It pays 3.95% (say) to A

Swaps
(Comparative Advantage)

Now the swap is through a FI and we have the flow transactions from the point of view of A after A and B get in touch with the FI

- A agrees to pay 4.0% to the outside agency
- A agrees to pay LIBOR (say) to FI
- A agrees to get 3.93% (say) from FI

Swaps
(Comparative Advantage)

Now the swap is through a FI and we have the flow transactions from the point of view of B after A and B get in touch with the FI

- B agrees to pay LIBOR+1.0% to the outside agency
- B agrees to pay 3.97% (say) to FI
- B agrees to get LIBOR (say) from FI

Swaps
(Comparative Advantage)

Now the swap is through a FI and we have the flow transactions from the point of view of FI after A and B get in touch with the FI

- FI agrees to pay 3.93% (say) to A
- FI agrees to get LIBOR (say) from A
- FI agrees to pay LIBOR (say) to B
- FI agrees to get 3.97% (say) from B

Example 5.1

Companies X wishes to borrow USD at a fixed rate of interest. Company Y wishes to borrow JPY at a fixed. The amounts required by the companies are roughly the same at the current exchange rate. The companies have been quoted the following interest rates, which have been adjusted for the impact of taxes

	JPY	USD
Company X	5.0%	9.6%
Company Y	6.5%	10.0%

Design a swap that will net a bank, acting as an intermediary, 50 basis points per annum. Make the swap equally attractive to the two companies and ensure that all foreign exchange risk is assumed by the bank

Example 5.2

Companies A and B have been offered the following rates per annum on a USD 20 million five year loan

	Fixed rate	Floating rate
Company A	12.0%	LIBOR+0.1%
Company B	13.4%	LIBOR+0.6%

Company A requires a floating rate loan, while company requires a fixed rate of loan. Design a swap that will net a bank, acting as an intermediary, 0.1% per annum and that will appear equally attractive to both the companies

Chapter 7:Properties of stock option prices

We will cover and discuss about

- Factors affecting prices
- Put-Call parity
- Call and put on non-dividend paying stock

Factors affecting option prices

- Current stock price
- Strike price
- Time to expiration
- Volatility of the stock price
- Risk free interest rate
- Dividends expected during the life of the option price

Factors affecting option prices

Var.	Eur.Call	Eur.Put	Am.Call	Am.Put
Stock price	+	+	-	-
Strike price	-	+	-	+
Time to expiry	?	?	+	+
Volatility	+	+	+	+
Risk free interest	+	-	+	-
Dividends	-	+	-	+

Factors affecting option prices

Few important assumptions

- There are no transactions cost
- All trading profits are subject to the same tax rate, i.e., the tax rate is fixed for all levels of income
- Borrowing and lending are possible at the same risk free interest rate

Few important notations to find option price

S_0: Current stock price

S_T: Stock price at time T

X: Strike price of option

T: Time to expiration of the option

r: Risk free interest rate

C: Value of American call option to buy one share

P: Value of American put option to sell one share

c: Value of European call option to buy one share

p: Value of European put option to sell one share

Upper and Lower bounds

- $C \leq S_0$
- $c \leq S_0$
- $P \leq X$
- $p \leq X$ to be more precise $p \leq Xe^{-rT}$

Lower bounds for European calls

For non-dividend paying stocks the lower bound is S_0-Xe^{-rT}

Let us consider an example where we have

- Portfolio A has one European call option plus an amount of Xe^{-rT} cash, which at time T is worth max(S_T,X)
- Portfolio B has one share, which at time T is worth S_T
- Hence $c \geq \max(S_0 - Xe^{-rT}, 0)$

Lower bounds for European puts

For non-dividend paying stocks the lower bound is $Xe^{-rT} - S_0$

Let us consider an example where we have

- Portfolio C has one European put option plus one share, which at time T is worth $\max(S_T, X)$
- Portfolio D has Xe^{-rT} amount of cash, which at time T is worth X
- Hence $p \geq \max(Xe^{-rT} - S_0, 0)$

Put-Call parity

Let us consider an example where we have

- Portfolio A has one European call option plus amount of cash equal to Xe^{-rT}
- Portfolio C has one European put option plus one share
- Hence $c + Xe^{-rT} = p + S_0$

Calls on non-dividend paying stock

Early exercise

- $c \geq S_0 - Xe^{-rT}$
- $C \geq c$
- $C \geq S_0 - Xe^{-rT}$

Puts on non-dividend paying stock

Early exercise

- $p \geq Xe^{-rT} - S_0$
- $P \geq X - S_0$

Effect of dividends

To find the lower bound for call, consider the simple example

- Portfolio A has one European call option plus an amount of cash equal to $DV+Xe^{-rT}$
- Portfolio B has one share
- Hence $c \geq S_0 - DV - Xe^{-rT}$

Effect of dividends

To find the lower bound for put, consider the simple example

- Portfolio C has one European put option plus one share
- Portfolio D has an amount of cash equal to $DV + Xe^{-rT}$
- Hence $p \geq DV + Xe^{-rT} - S_0$

Effect of dividends

- Put-Call parity is

$$c + DV + Xe^{-rT} = p + S_0$$

- Relation between American put and call prices is

$$S_0 - DV - X \leq C - P \leq S_0 - Xe^{-rT}$$

Example 7.1

What is the lower bound for the price of a four month call option on a non-dividend paying stock when the stock price is $28, the strike price is $25, and the risk free interest rate is 8% per annum

Example 7.2

A four month European call option on a dividend paying stock is currently selling for $5. The stock price is $64, the strike price $60 and a dividend of $0.80 is expected in one month. The risk free interest rate is 12% per annum for all maturities. What opportunities are there for an arbitrageur?

Chapter 8:Trading strategies involving derivatives

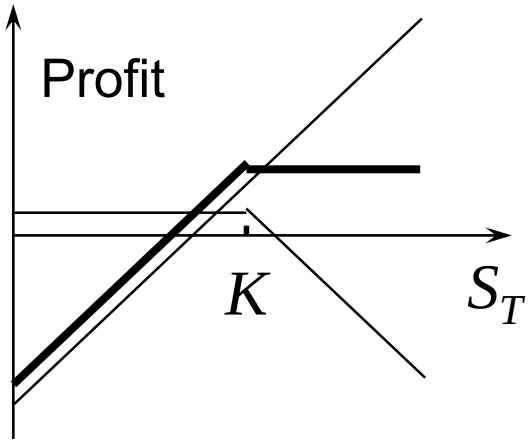
We will cover and discuss about

- Combination of forward and an option
- Combination of call/put options

Trading Strategies Involving Options

- In the first chapter we had discussed the payoffs/profits from an investment in a SINGLE call/put option. The question is what happens to the payoff/profit patterns using different options simultaneously.

Positions in an Option & the Underlying (which we will take as the stock)



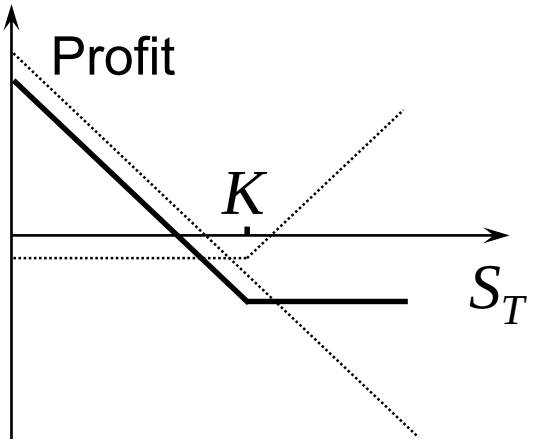
- ***Long*** position in a ***Stock***
- ***Short*** position in a ***Call Option***

Positions in an Option & the Underlying (which we will take as the stock)

This transaction is know as writing a covered call. In the sense that it protects or covers the investor from the payoff on the short call that becomes necessary if there is a sharp rise in the stock price

Positions in an Option & the Underlying (which we will take as the stock)

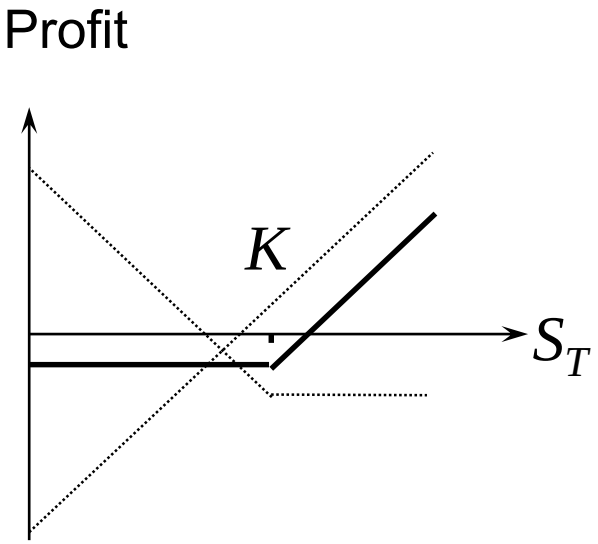
- ***Short*** position in a ***Stock***
- ***Long*** position in a ***Call Option***



Positions in an Option & the Underlying (which we will take as the stock)

This transaction is just the reverse of writing a covered call

Positions in an Option & the Underlying (which we will take as the stock)



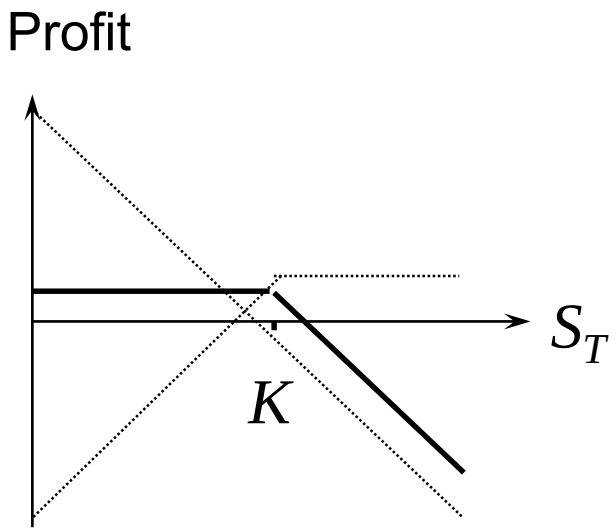
- ***Long*** position in a ***Stock***
- ***Long*** position in a ***Put Option***

Positions in an Option & the Underlying (which we will take as the stock)

This transaction is called the protective put

Positions in an Option & the Underlying (which we will take as the stock)

- ***Short*** position in a Stock
- ***Short*** position in a ***Put Option***



Positions in an Option & the Underlying (which we will take as the stock)

This transaction is just the reverse of the protective put

Spreads

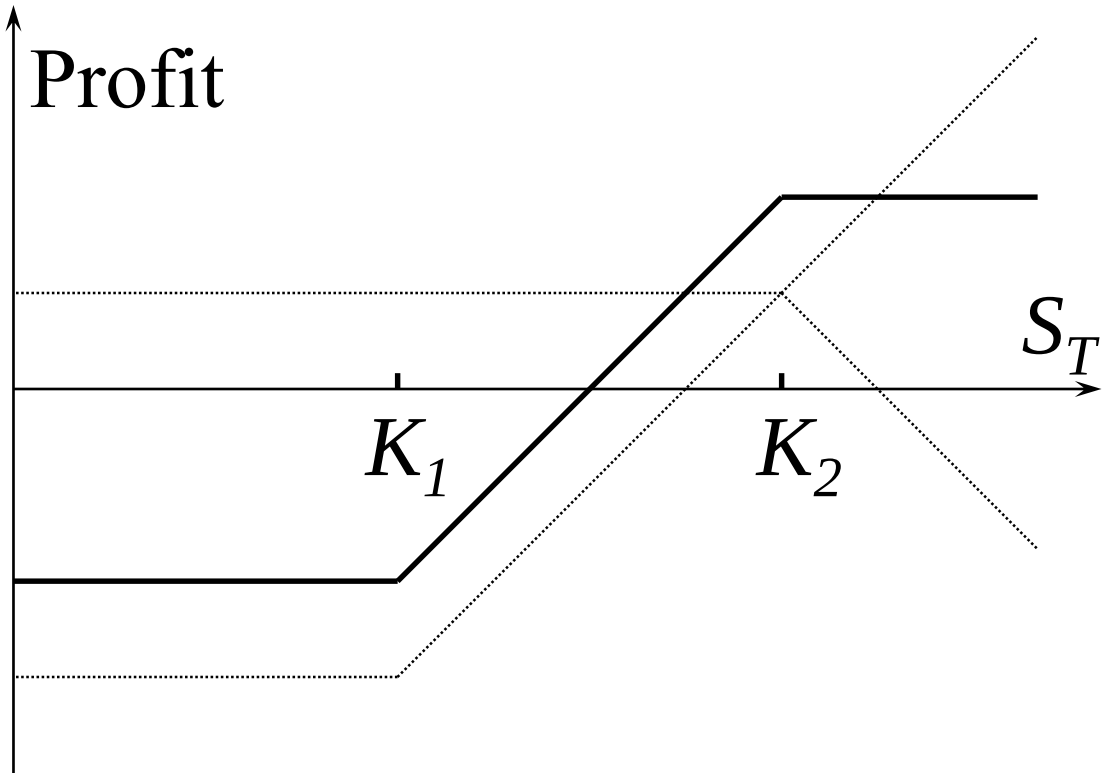
A Spread strategy involves taking a position in TWO or MORE OPTIONS at the SAME TIME. Some examples are

- Bull Spreads
- Bear Spreads
- Butterfly Spreads

Spreads

Bull Spreads: The investor is hoping that the stock price will **increase**

Bull Spread Using Calls



Bull Spread Using Calls

S_T	LC	SC	Net profit
$S_T \le K_1$	$-c_{LC}$	$+c_{SC}$	$-c_{LC}+c_{SC}$
$K_1 < S_T \le K_2$	$S_T-K_1-c_{LC}$	$+c_{SC}$	$S_T-K_1-c_{LC}+c_{SC}$
$S_T > K_2$	$S_T-K_1-c_{LC}$	$K_2-S_T+c_{SC}$	$S_T-K_1-c_{LC}+K_2-S_T+c_{SC}$

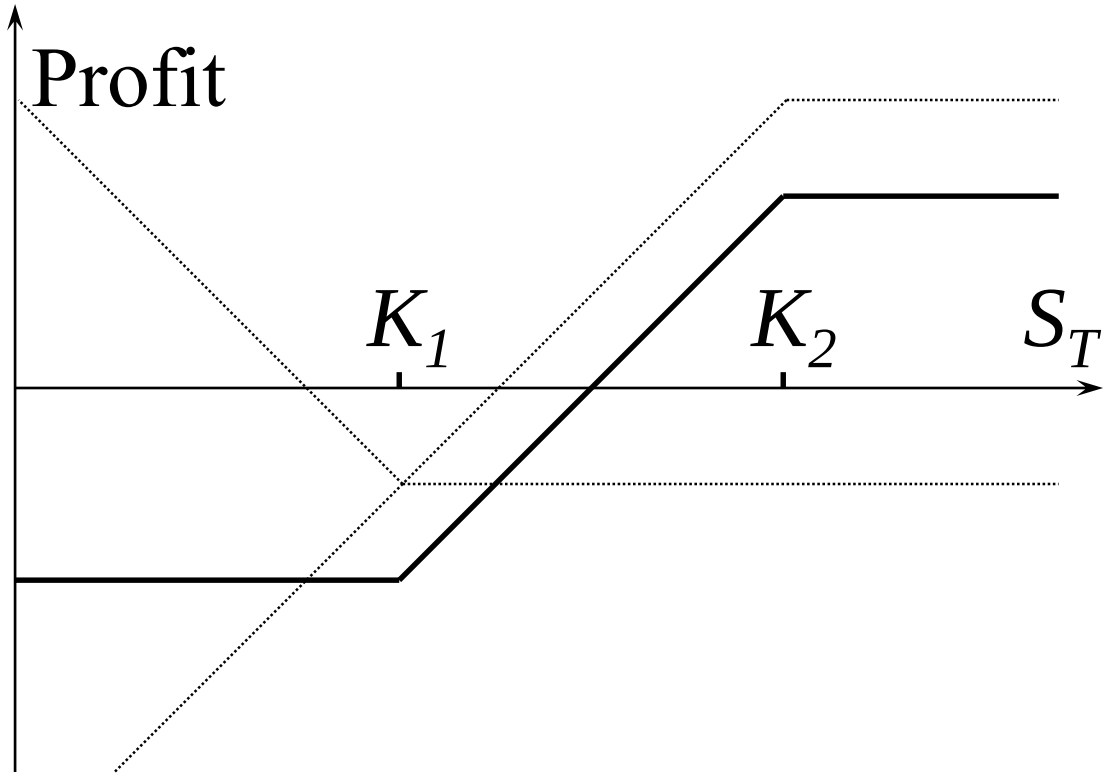
Bull Spread Using Calls

Consider you have simultaneously gone into a long call and short call for the same maturity (i.e., T is same). We have the following information, LC ($K_1 = 100$, $c_{LC} = -5$) and SC ($K_2 = 150$, $c_{SC} = +8$). Solve this problem. First make the payoff matrix and then draw the cumulative payoff diagram (in excel)

Bull Spread Using Calls

S_T	LC	SC	Net profit
$S_T \leq K_1$	$-c_{LC}$	$+c_{SC}$	$-c_{LC}+c_{SC}$
$K_1 < S_T \leq K_2$	$S_T-K_1-c_{LC}$	$+c_{SC}$	$S_T-K_1-c_{LC}+c_{SC}$
$S_T > K_2$	$S_T-K_1-c_{LC}$	$K_2-S_T+c_{SC}$	$S_T-K_1-c_{LC}+K_2-S_T+c_{SC}$

Bull Spread Using Puts



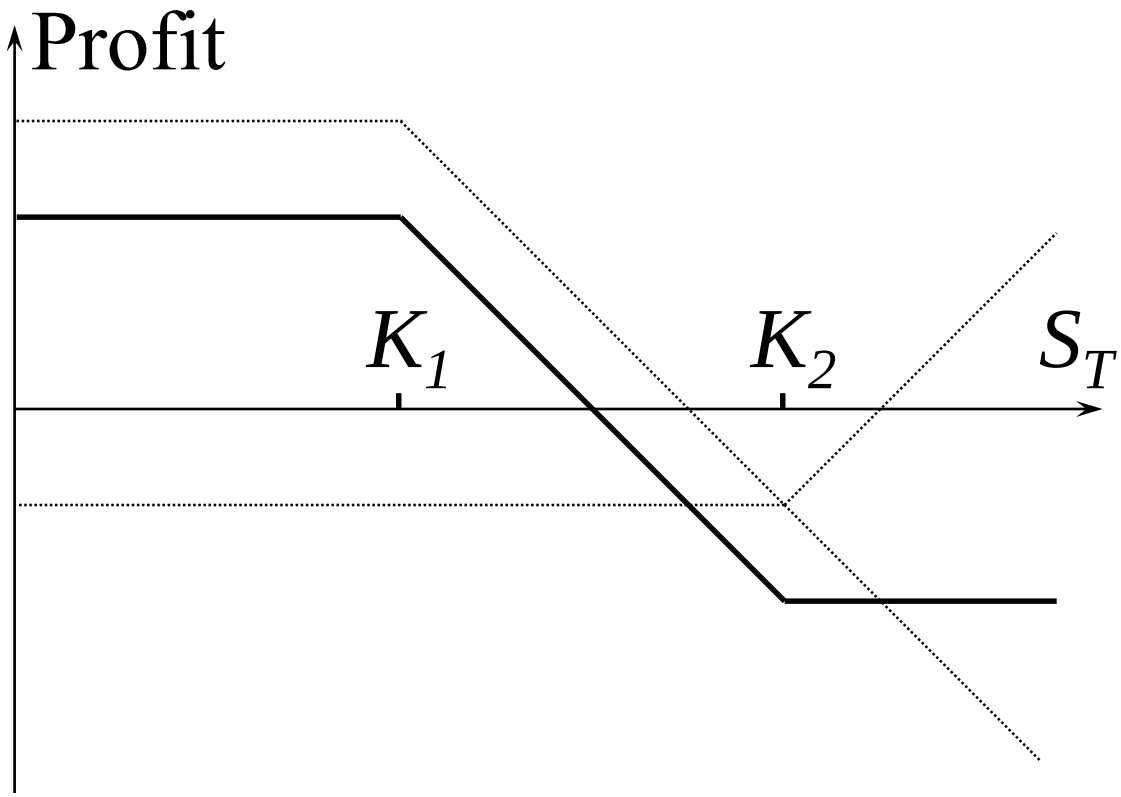
Bull Spread Using Puts

S_T	LP	SP	Net profit
$S_T \leq K_1$	$K_1 - S_T - p_{LP}$	$S_T - K_2 + p_{SP}$	$K_1 - S_T - p_{LP} + S_T - K_2 + p_{SP}$
$K_1 < S_T \leq K_2$	$-p_{LP}$	$S_T - K_2 + p_{SP}$	$-p_{LP} + S_T - K_2 + p_{SP}$
$S_T > K_2$	$-p_{LP}$	$+p_{SP}$	$-p_{LP} + p_{SP}$

Spreads

Bear Spreads: The investor is hoping that the stock price will **decrease**

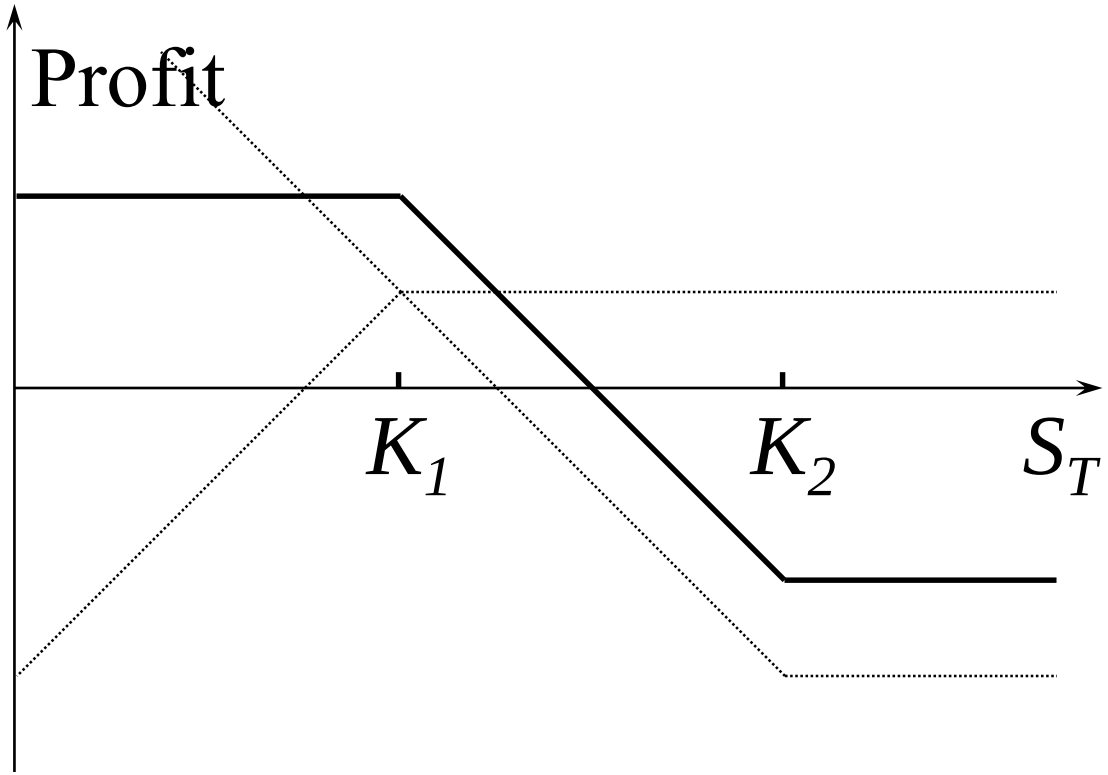
Bear Spread Using Calls



Bear Spread Using Calls

S_T	SC	LC	Net profit
$S_T \leq K_1$	$+c_{SC}$	$-c_{LC}$	$+c_{SC}-c_{LC}$
$K_1 < S_T \leq K_2$	$K_1-S_T+c_{SC}$	$-c_{LC}$	$K_1-S_T+c_{SC}-c_{LC}$
$S_T > K_2$	$K_1-S_T+c_{SC}$	$S_T-K_2-c_{LC}$	$K_1-S_T+c_{SC}+S_T-K_2-c_{LC}$

Bear Spread Using Puts



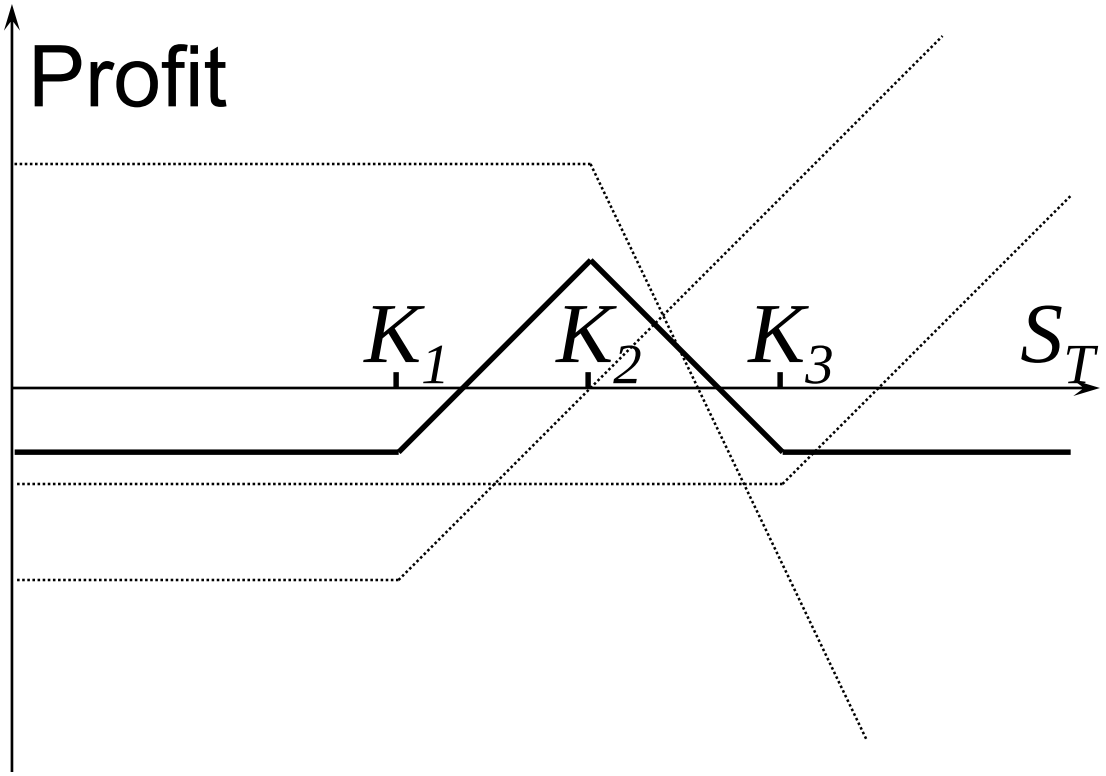
Bear Spread Using Puts

S_T	SP	LP	Net profit
$S_T \leq K_1$	$S_T - K_1 + p_{SP}$	$K_2 - S_T - p_{LP}$	$S_T - K_1 + p_{SP} + K_2 - S_T - p_{LP}$
$K_1 < S_T \leq K_2$	$+p_{SP}$	$K_2 - S_T - p_{LP}$	$+p_{SP} + K_2 - S_T - p_{LP}$
$S_T > K_2$	$+p_{SP}$	$-p_{LP}$	$+p_{SP} - p_{LP}$

Spreads

Butterfly Spreads: The investor is hoping that large stock price movements are unlikely

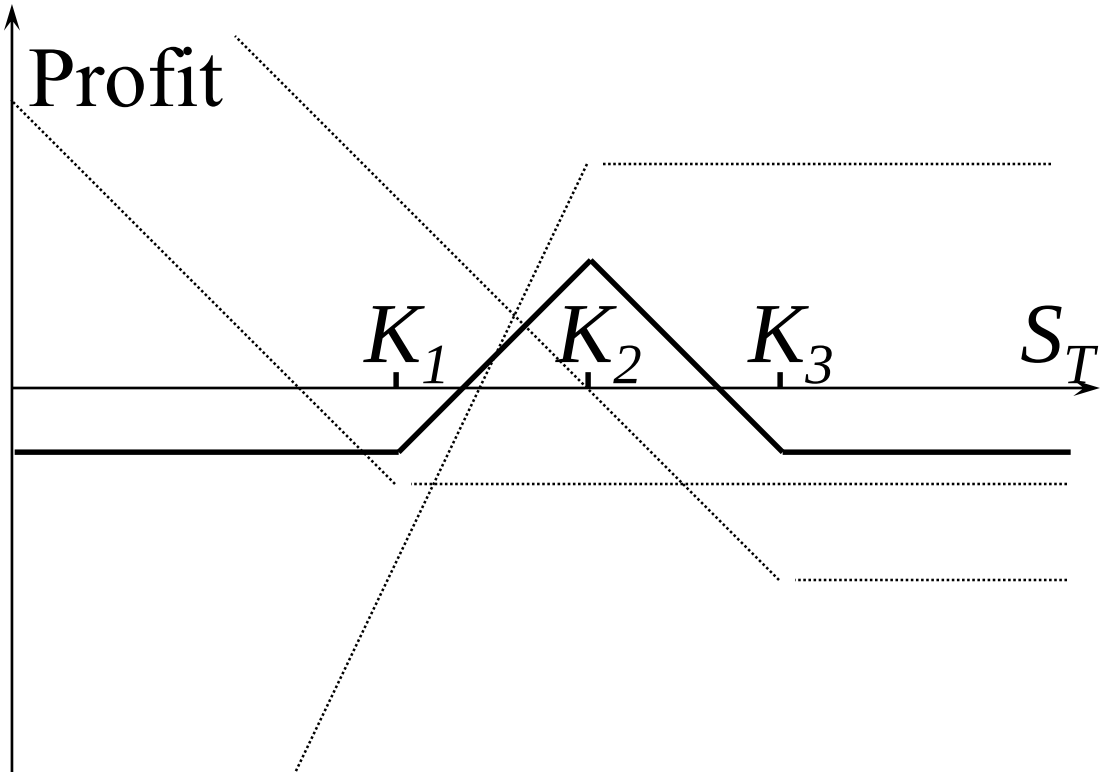
Butterfly Spread Using Calls



Butterfly Spread Using Calls

S_T	LC	SC	LC
$S_T \leq K_1$	$-c_{LC1}$	$+c_{SC}$	$-c_{LC2}$
Hence profit = $-c_{LC1}+c_{SC}-c_{LC2}$			
$K_1 < S_T \leq K_2$	$S_T-K_1-c_{LC1}$	$+c_{SC}$	$-c_{LC2}$
Hence profit = $S_T-K_1-c_{LC1}+c_{SC}-c_{LC2}$			
$K_2 < S_T \leq K_3$	$S_T-K_1-c_{LC1}$	$K_2-S_T+c_{SC}$	$-c_{LC2}$
Hence profit = $S_T-K_1-c_{LC1}+K_2-S_T+c_{SC}-c_{LC2}$			
$S_T > K_3$	$S_T-K_1-c_{LC1}$	$K_2-S_T+c_{SC}$	$S_T-K_3-c_{LC2}$
Hence profit = $S_T-K_1-c_{LC1}+K_2-S_T+c_{SC}+S_T-K_3-c_{LC2}$			

Butterfly Spread Using Puts



Butterfly Spread Using Puts

S_T	LP	SP	LP
$S_T \leq K_1$	$K_1+S_T-p_{LP1}$	$S_T-K_2+p_{SP}$	$K_3+S_T-p_{LP2}$

Hence profit = $K_1+S_T-p_{LP1}+ S_T-K_2+p_{SP}+K_3+S_T-p_{LP2}$

$K_1<S_T \leq K_2$	$-p_{LP1}$	$S_T-K_2+p_{SP}$	$K_3+S_T-p_{LP2}$

Hence profit = $-p_{LP1}+S_T-K_2+p_{SP}+K_3+S_T-p_{LP2}$

$K_2<S_T \leq K_3$	$-p_{LP1}$	$+p_{SP}$	$K_3+S_T-p_{LP2}$

Hence profit = $-p_{LP1}+p_{SP}+K_3+S_T-p_{LP2}$

$S_T>K_3$	$-p_{LP1}$	$+p_{SP}$	$-p_{LP2}$

Hence profit = $-p_{LP1}+p_{SP}-p_{LP2}$

Trading Strategies

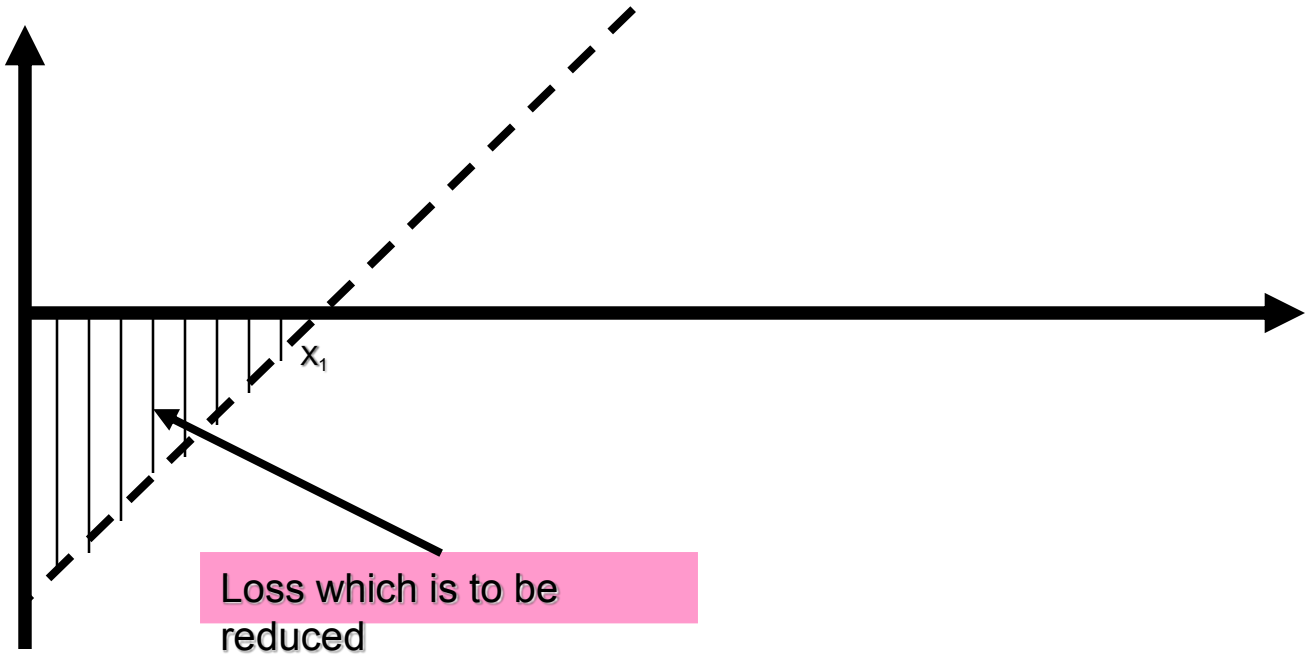
Trading strategies involving combinations of **forwards** and **options** in order to reduce risk

Trading Strategies

For the problems which we will discuss we denote the delivery price by X

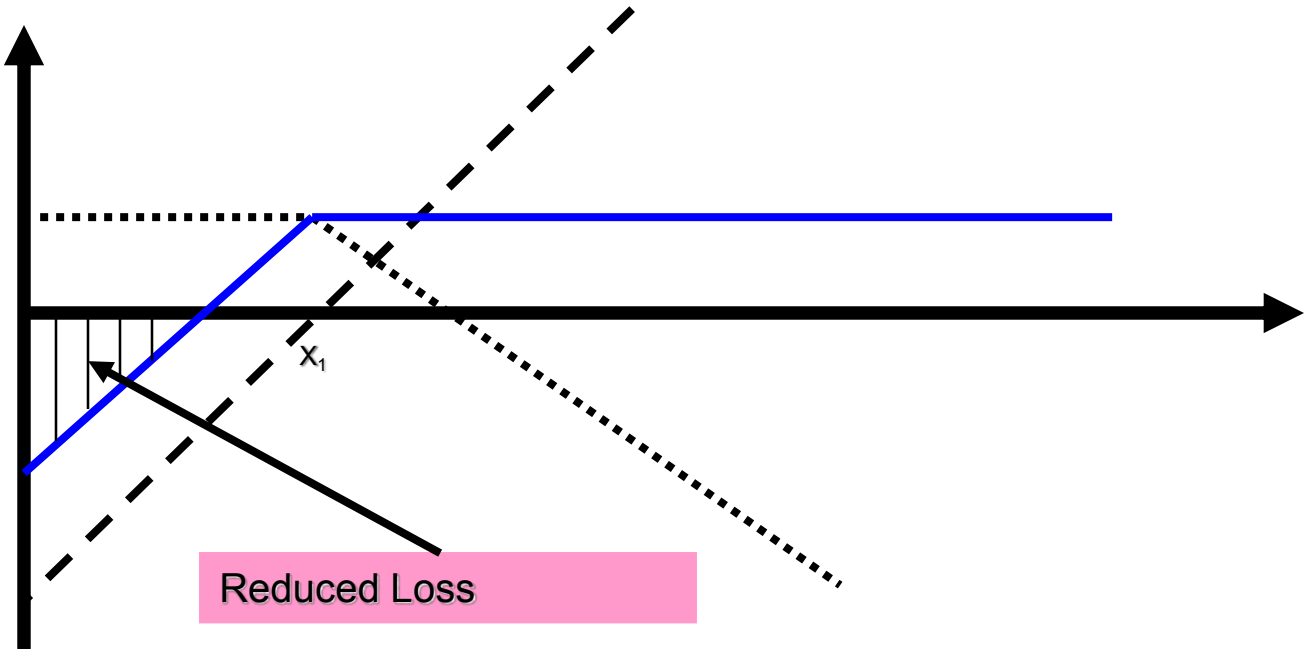
Long forward combined with a short call option

Initially we only have a long forward and the risk is as shown, considering prices will generally be on a **down trend**



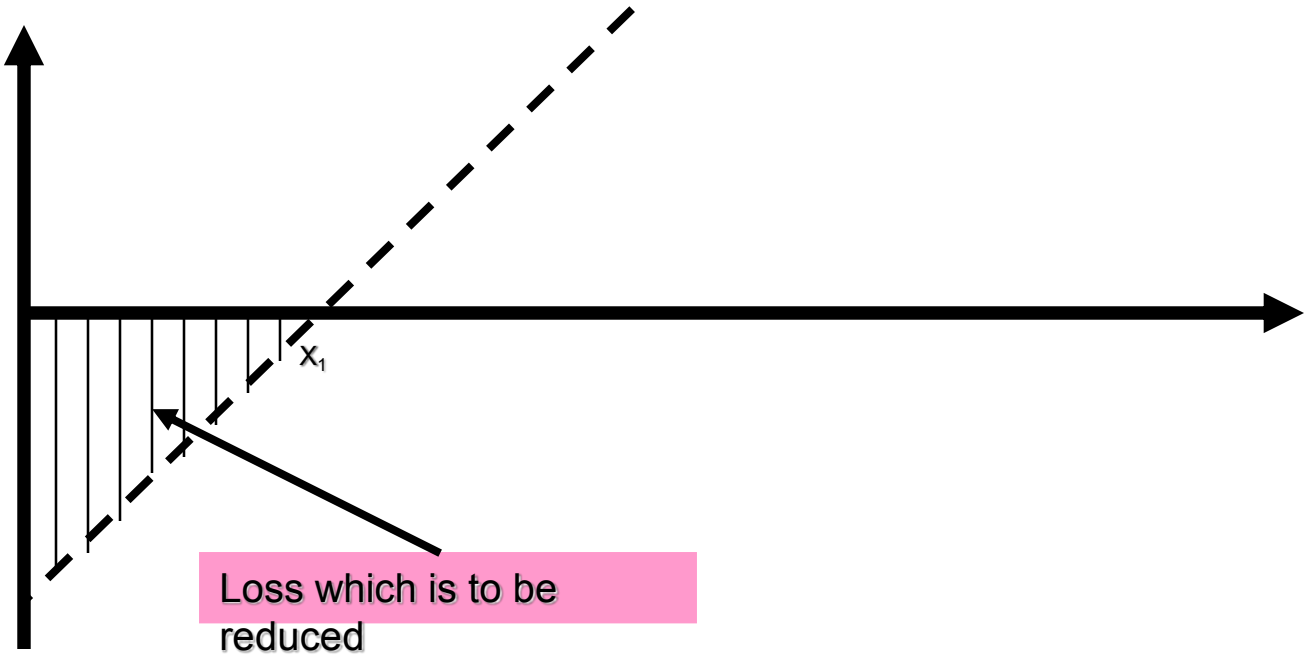
Long forward combined with a short call option

With the long forward we combine a short call option and the cumulative payoff (bold line) is shown



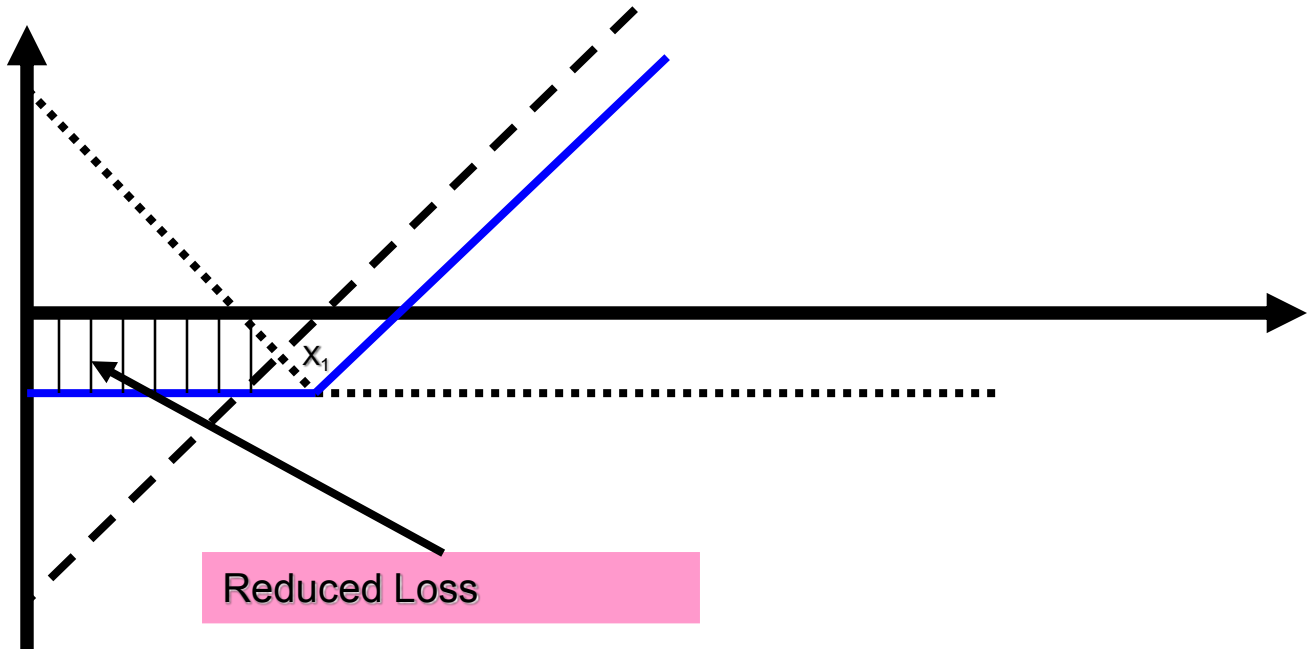
Long forward combined with a long put option

Initially we only have a long forward and the risk is as shown, considering prices will generally be on a **down trend**



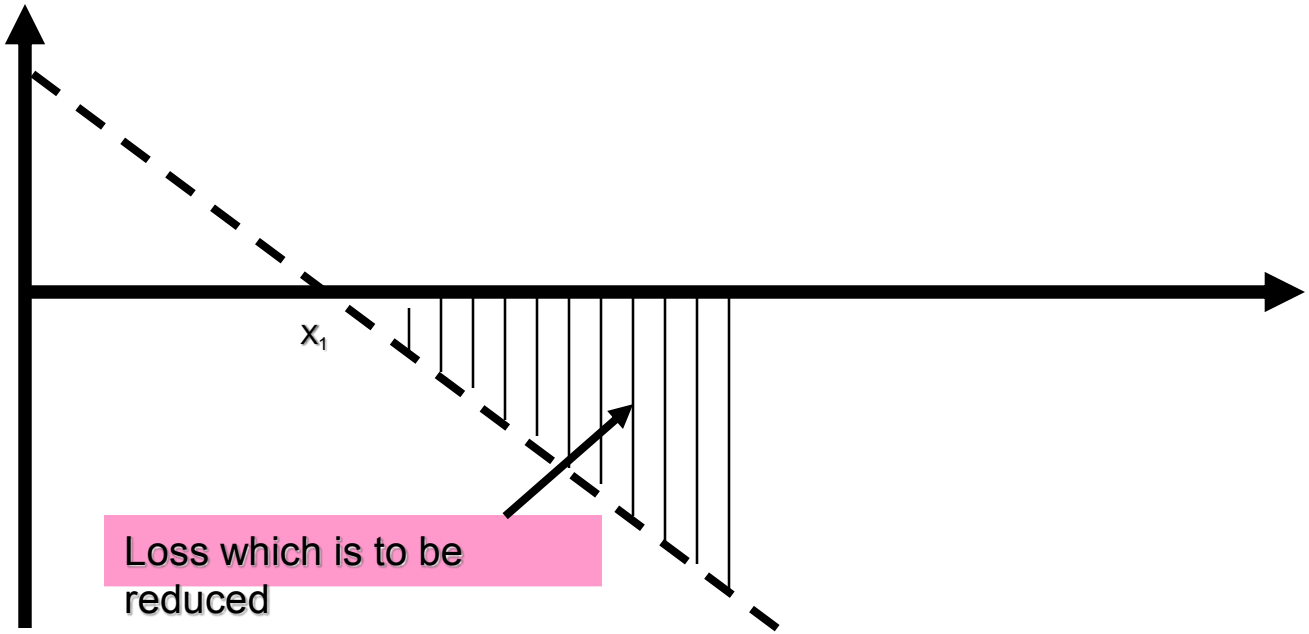
Long forward combined with a long put option

With the long forward we combine a long put option and the cumulative payoff (bold line) is shown



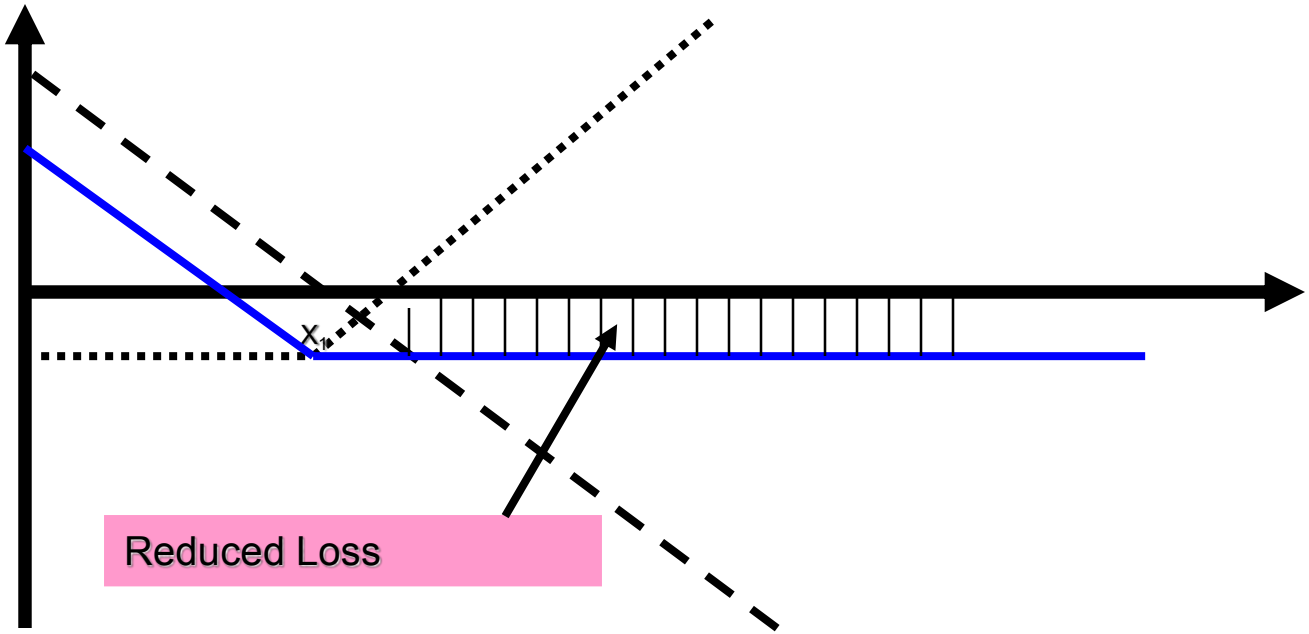
Short forward combined with a long call option

Initially we only have a short forward and the risk is as shown, considering prices will generally be on an **upward trend**



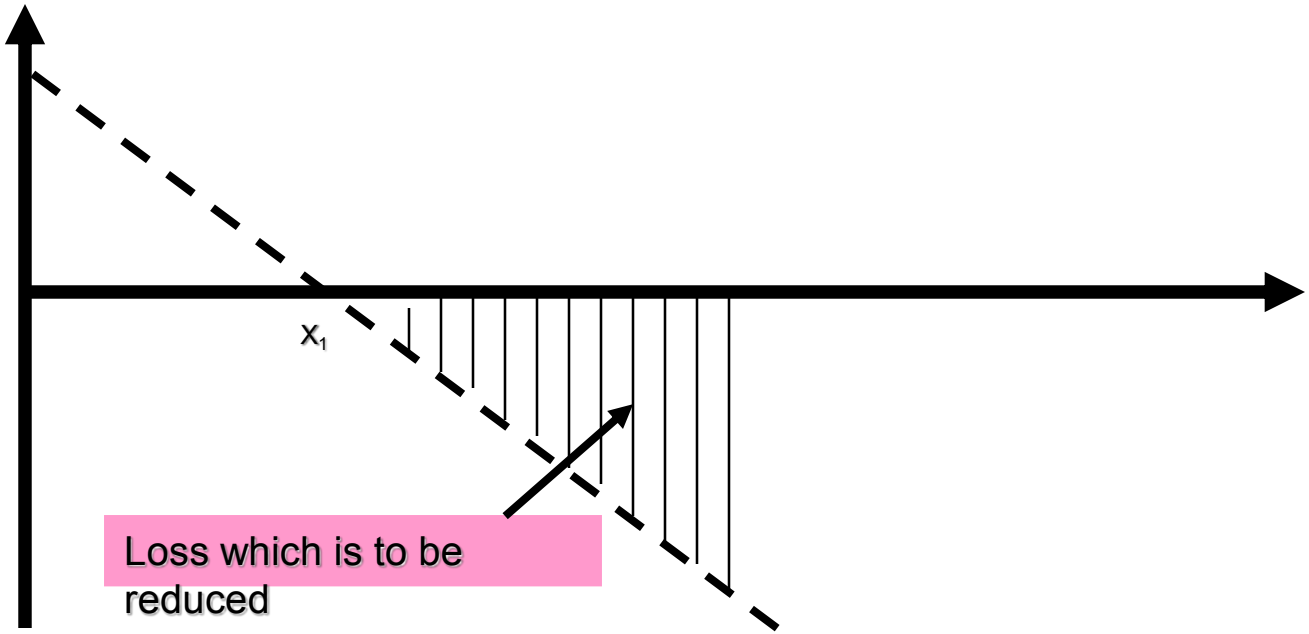
Short forward combined with a long call option

With the short forward we combine a long call option and the cumulative payoff (bold line) is shown



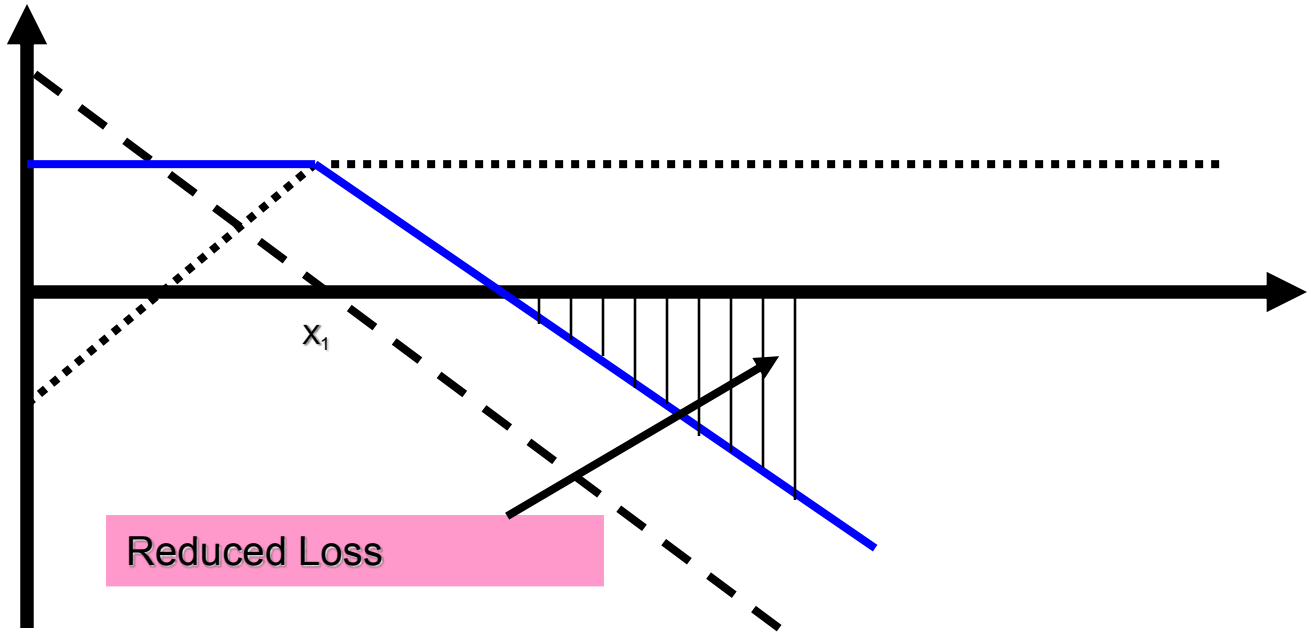
Short forward combined with a short put option

Initially we only have a short forward and the risk is as shown, considering prices will generally be on an **upward trend**



Short forward combined with a short put option

With the short forward we combine a short put option and the cumulative payoff (bold line) is shown



Trading strategies involving only two options

We consider the following

- Initially the person sells or buys an option (can be either put or call) and then for reasons (can be either due to an up ward or down ward movement of prices, i.e, S_T) he/she would like to minimize the loss, i.e., reduce the risk.
- X is the lower strike price.
- Y is the higher strike price.
- Some premium is paid/obtained for each option.

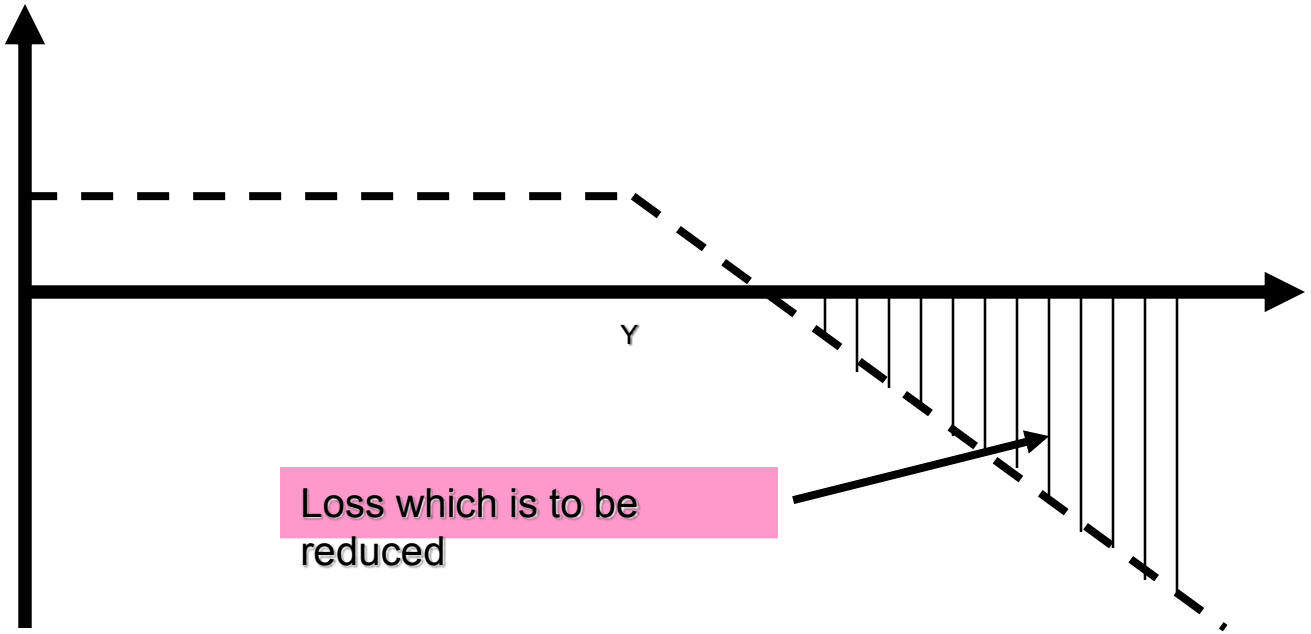
Trading strategies involving only two options

It is important to remember the following

- For a call option we need to take action when you speculate the price (S_T) would **rise** (i.e, the price is on an upward trend). As the main concern is to highlight how to **reduce loss/risk**, hence we consider examples where we **first start** with **a short call option** and then add other combinations of options to reduce risk/loss.
- For a put option we need to take action when you speculate the price (S_T) would **fall** (i.e, the price is on a downward trend). As the main concern is to highlight how to **reduce loss/risk**, hence we consider examples where we **first start** with **a short put option** and then add other combinations of options to reduce risk/loss.

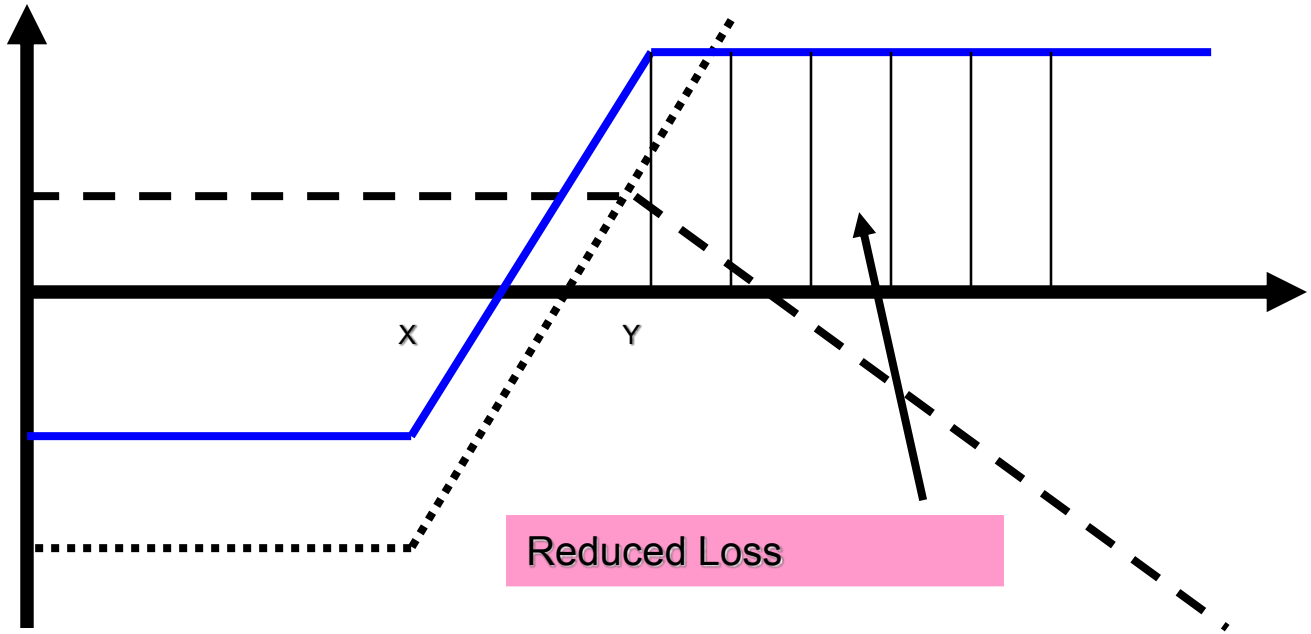
A **Short Call** is combined with a **Long Call**

Initially we have a short call option only and the risk is as shown, considering prices will generally be on an **upward trend**



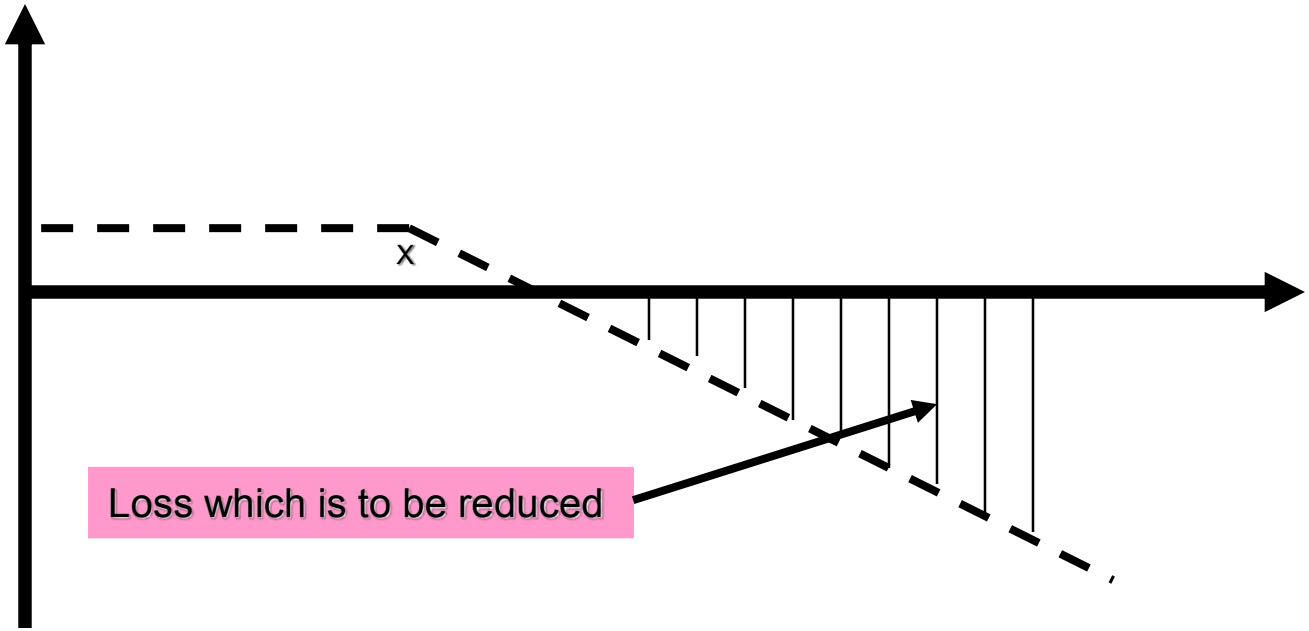
A **Short Call** is combined with a **Long Call**

With the short call option we combine a long call option (**lower delivery price**) and the cumulative payoff (bold line) is shown



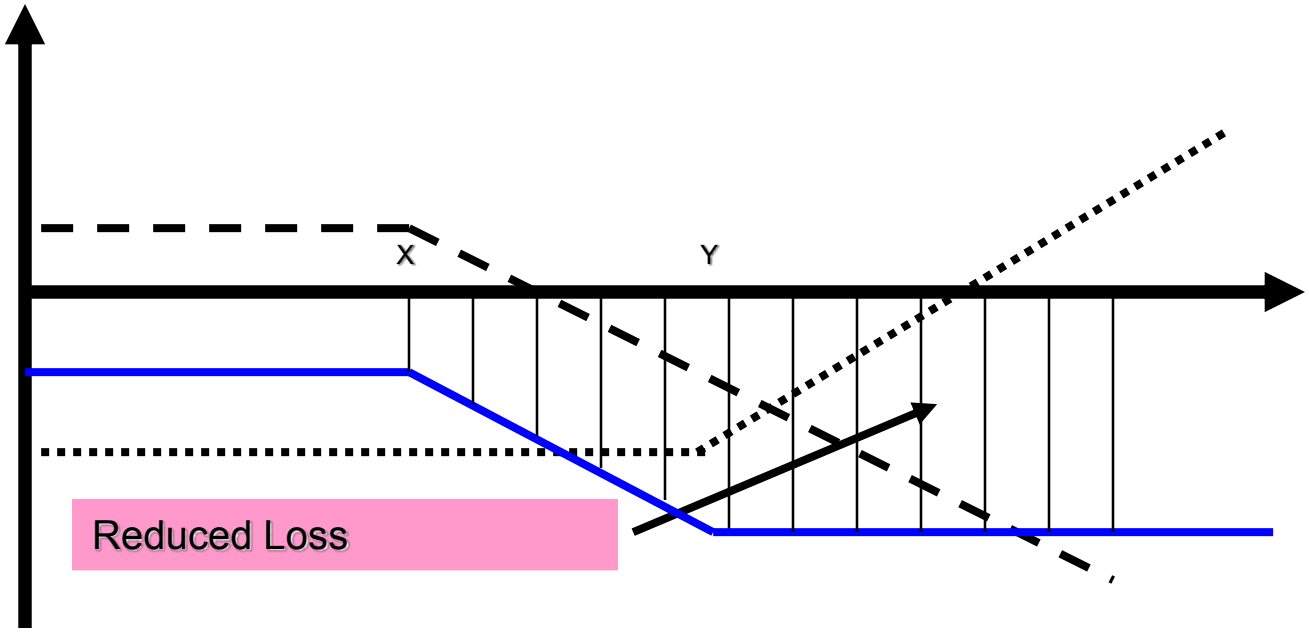
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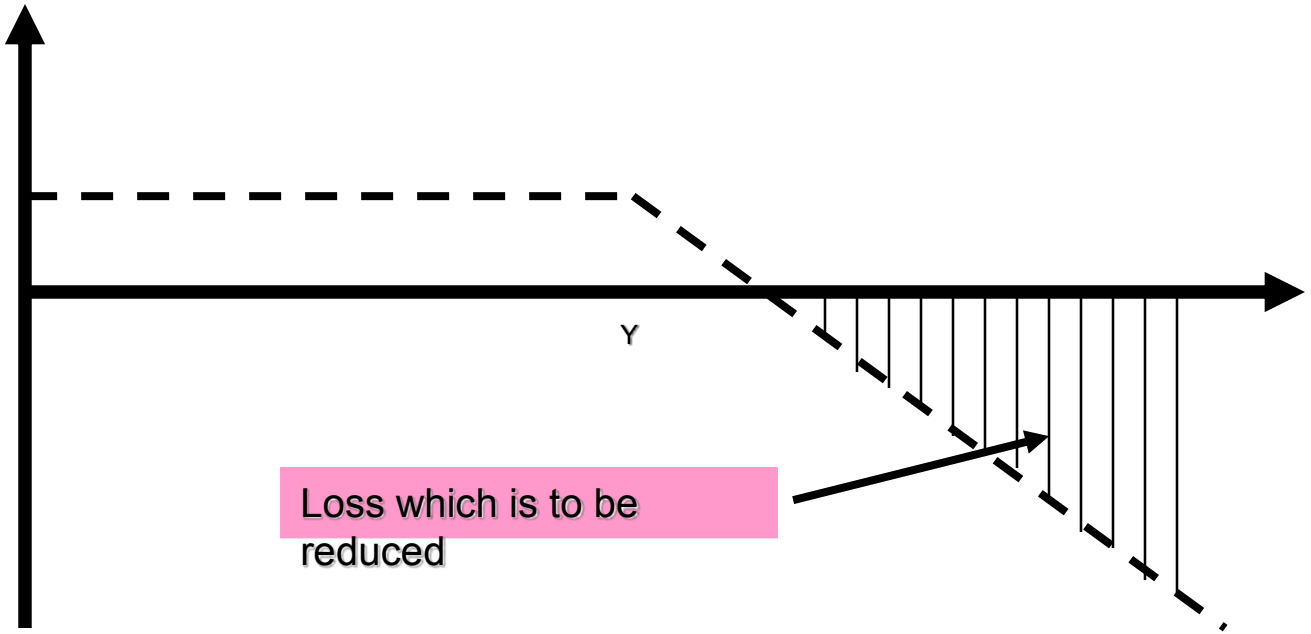
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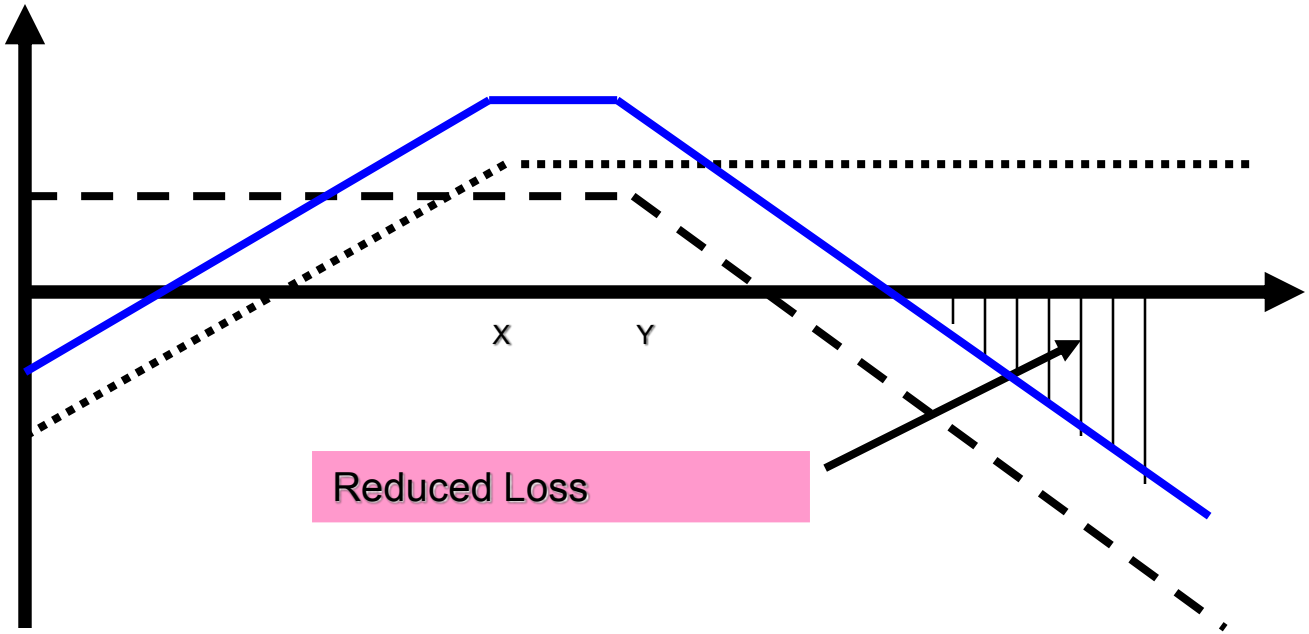
A **Short Call** is combined with a **Short Put**

Initially we have a short call option only and the risk is as shown, considering prices will generally be on an **upward trend**



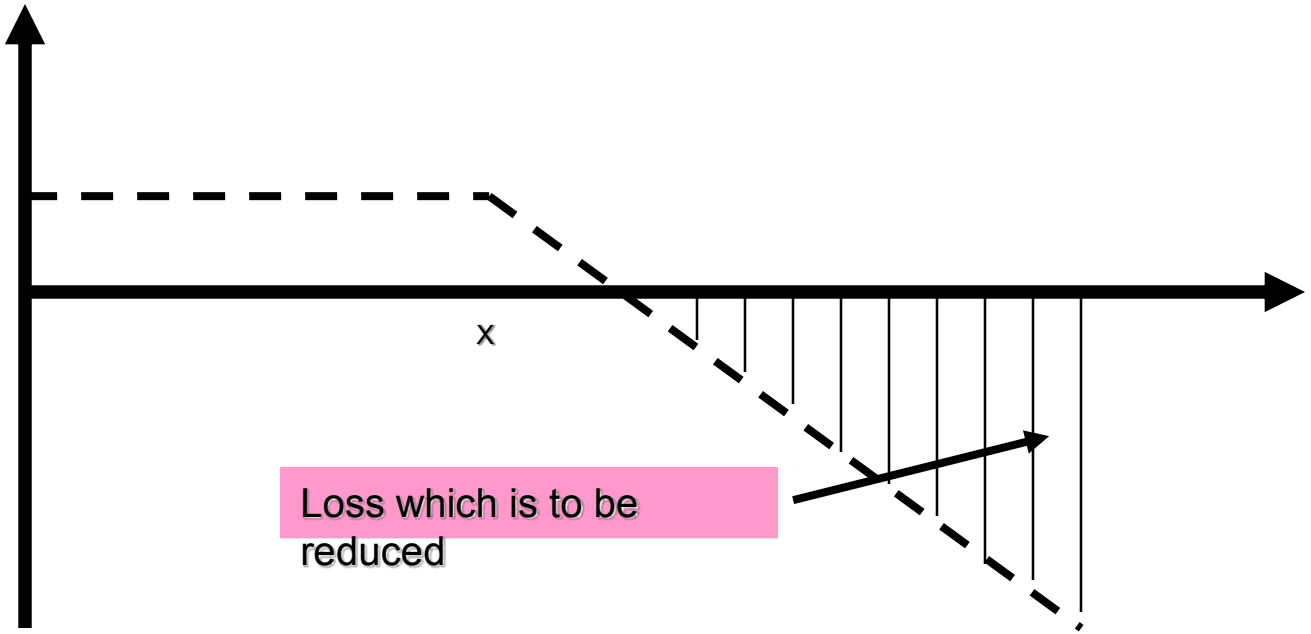
A **Short Call** is combined with a **Short Put**

With the short call option we combine a short put option (**lower delivery price**) and the cumulative payoff (bold line) is shown



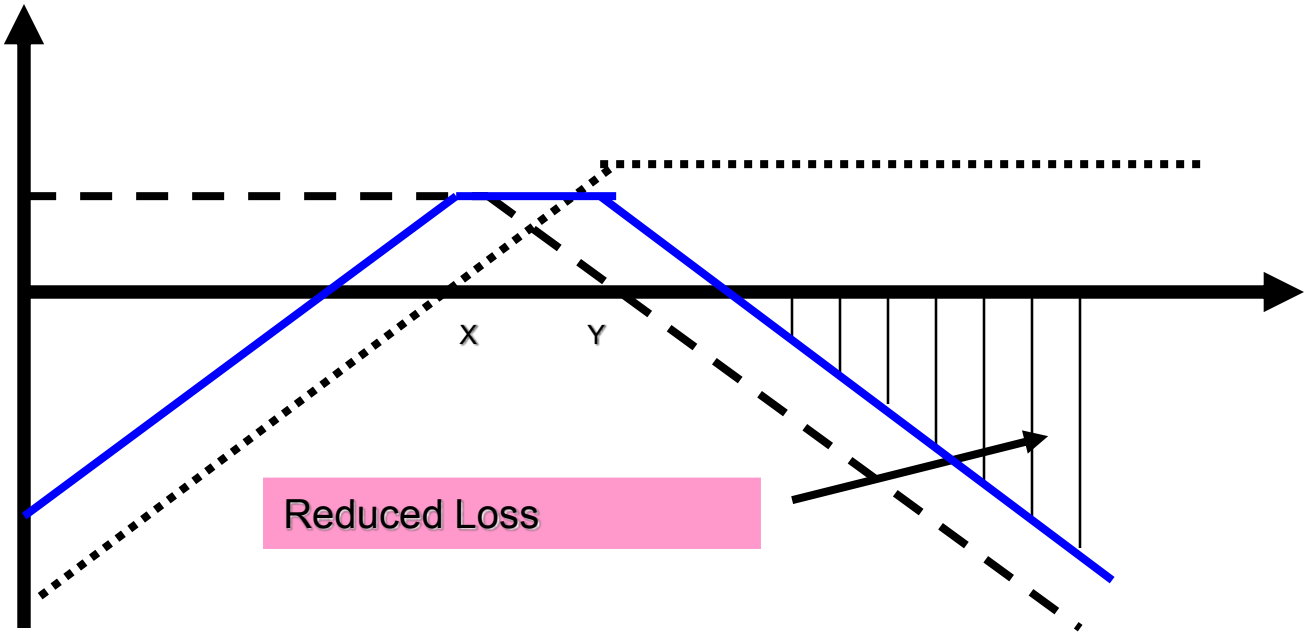
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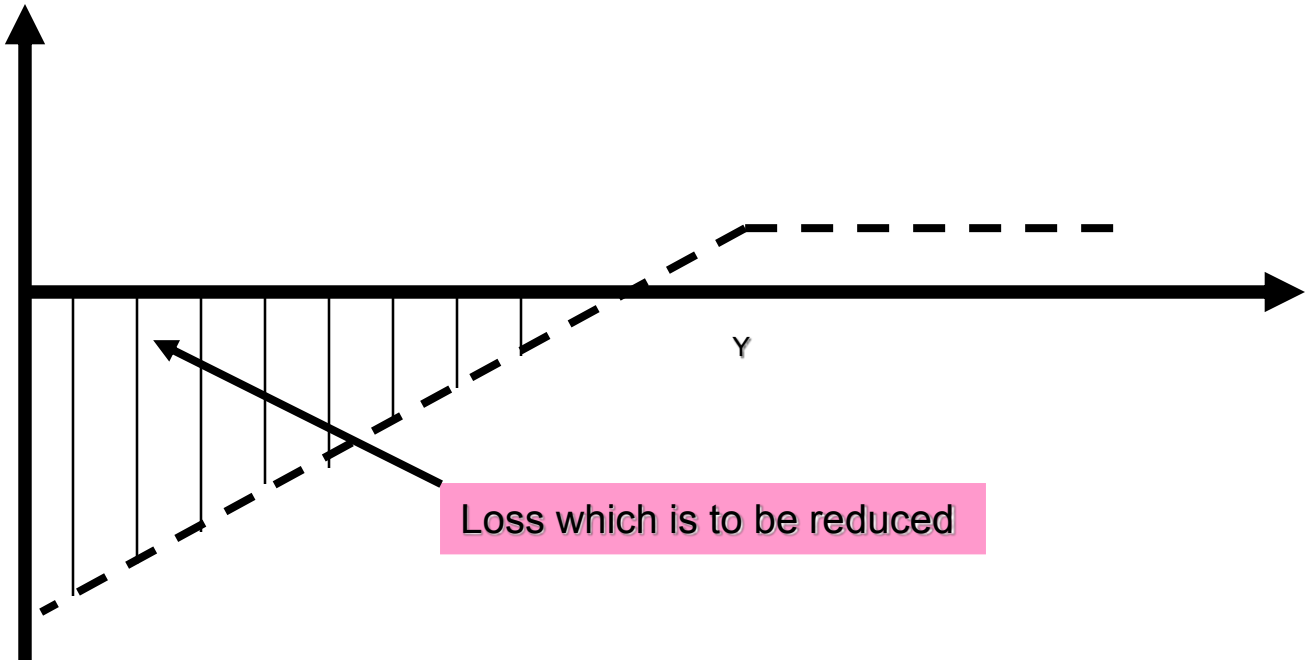
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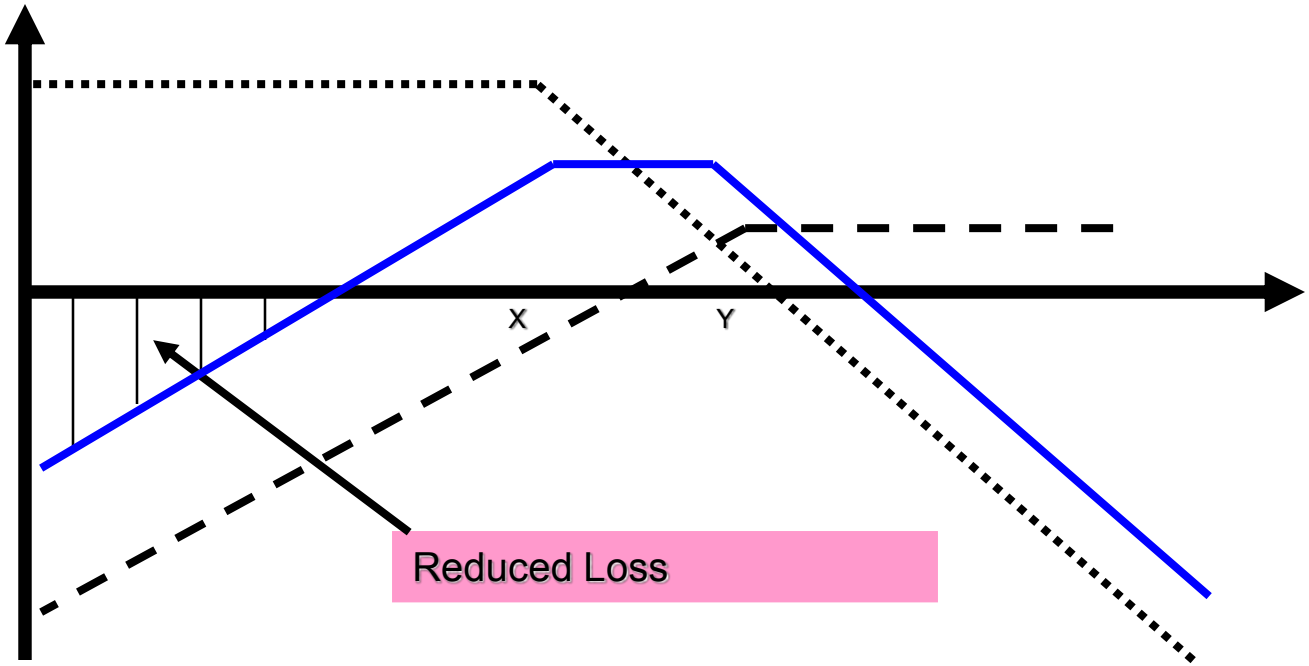
A **Short Put** is combined with a **Short Call**

Initially we have a short put option only and the risk is as shown, considering prices will generally be on a **downward trend**



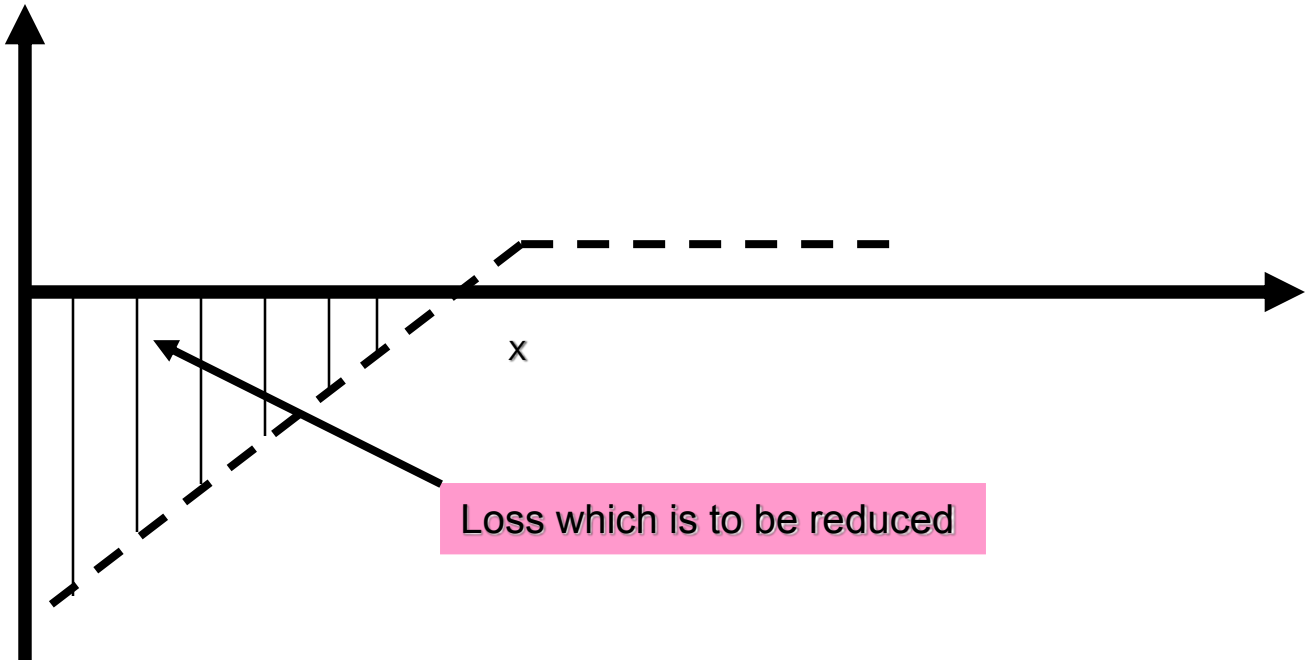
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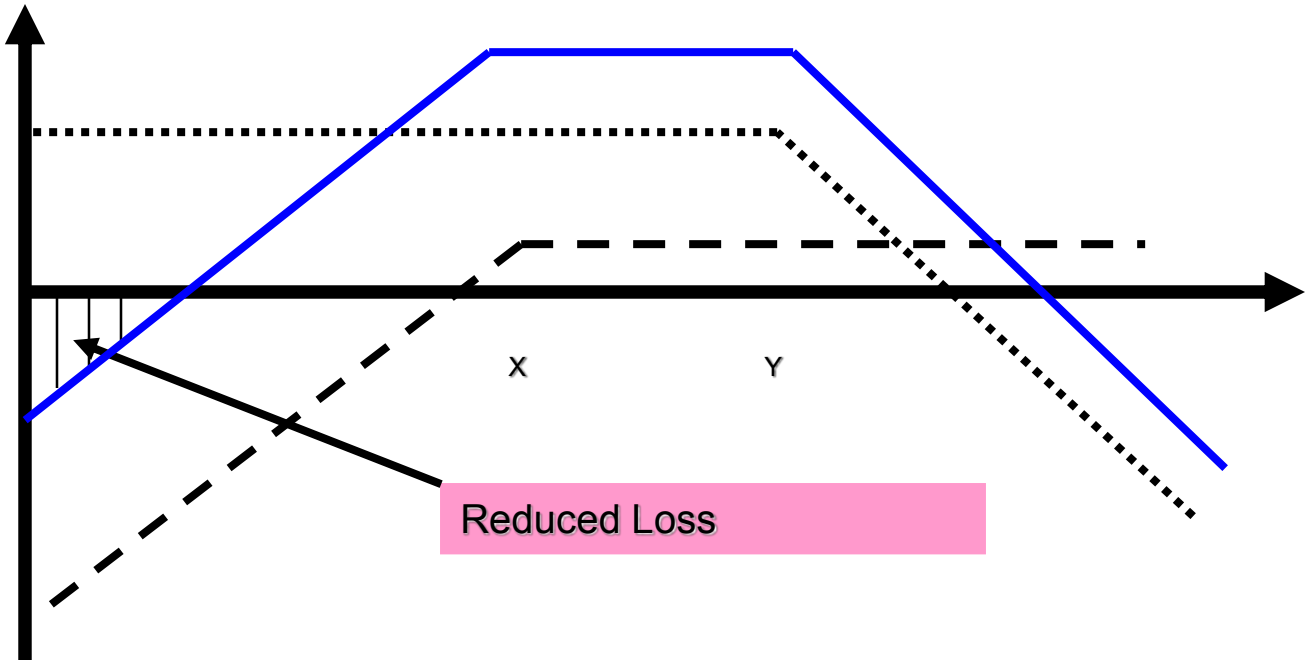
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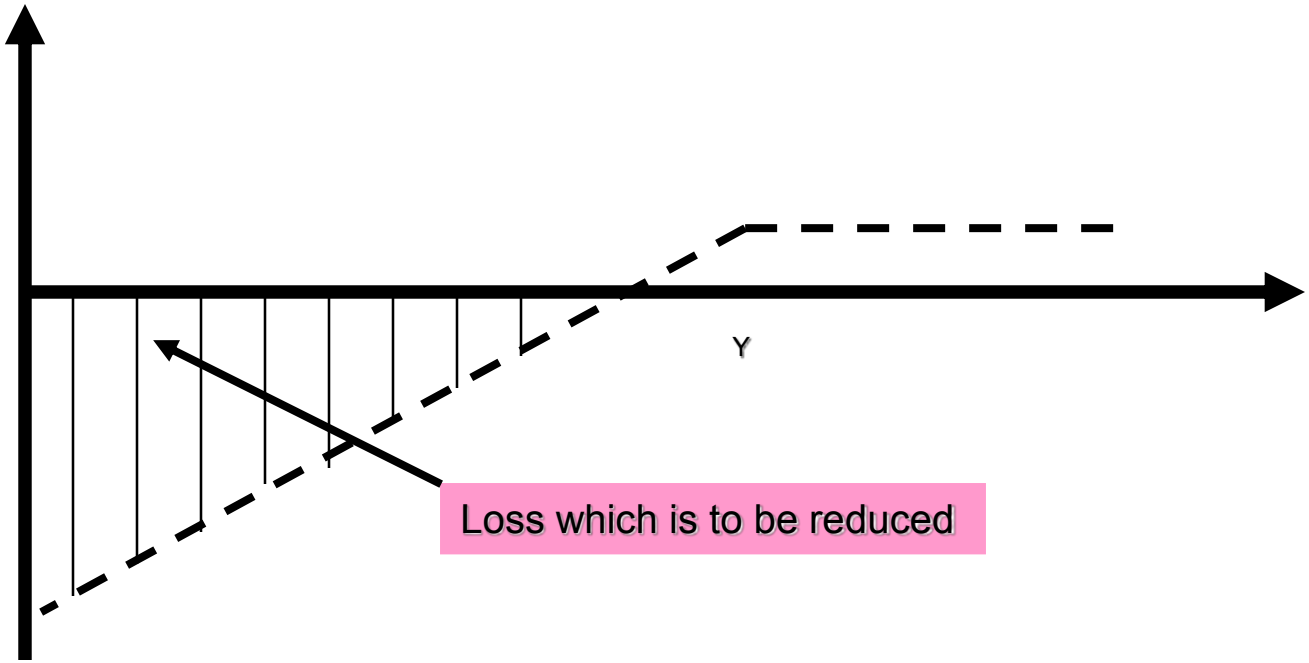
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With the short put option we combine a short call option (**higher delivery price**) and the cumulative payoff (bold line) is shown



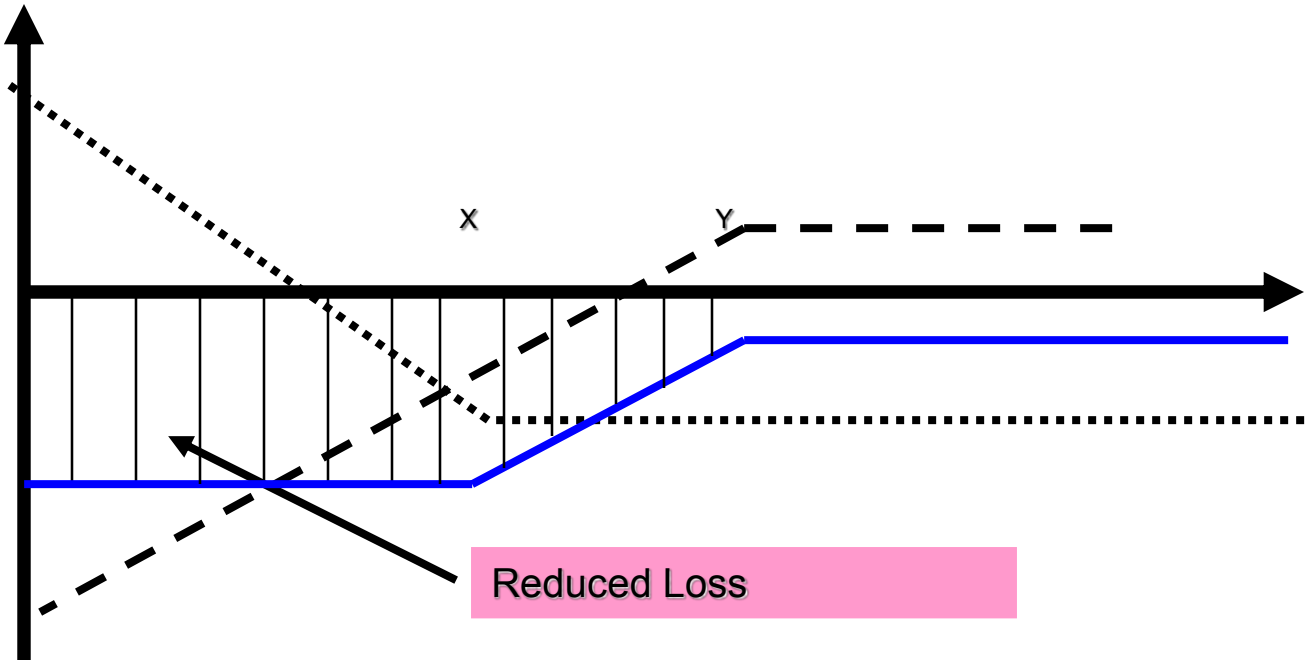
A **Short Put** is combined with a **Long Put**

Initially we have a short put option only and the risk is as shown, considering prices will generally be on a **downward trend**



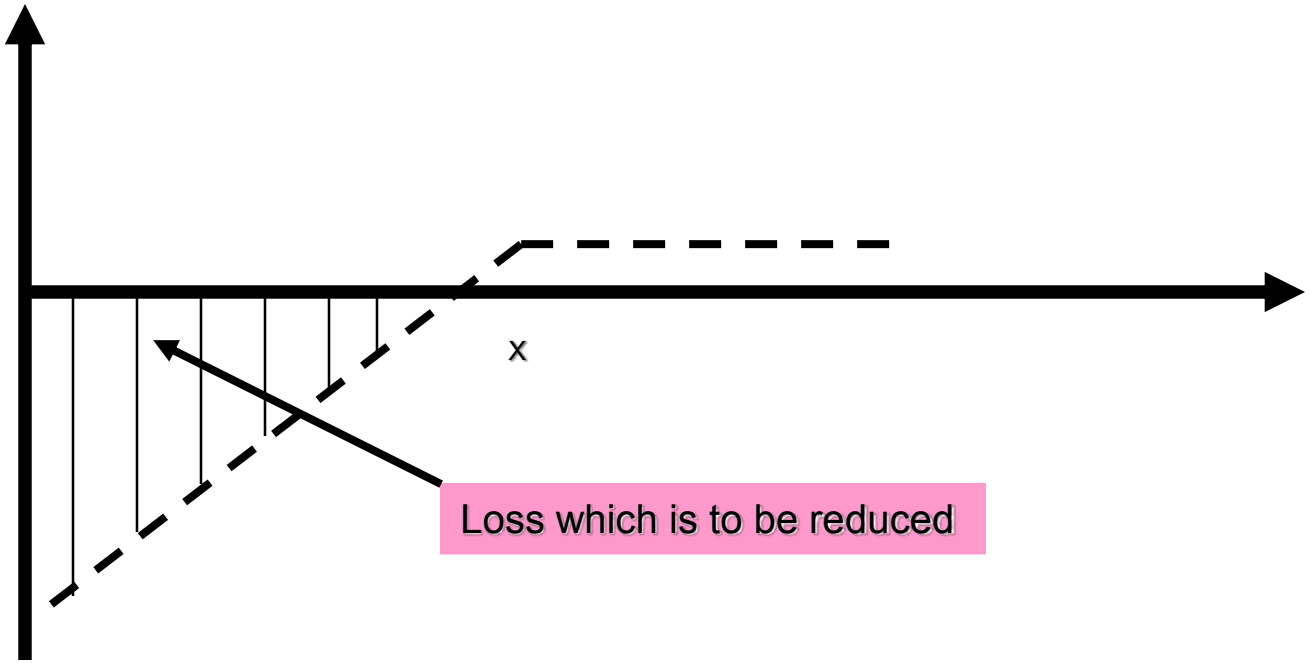
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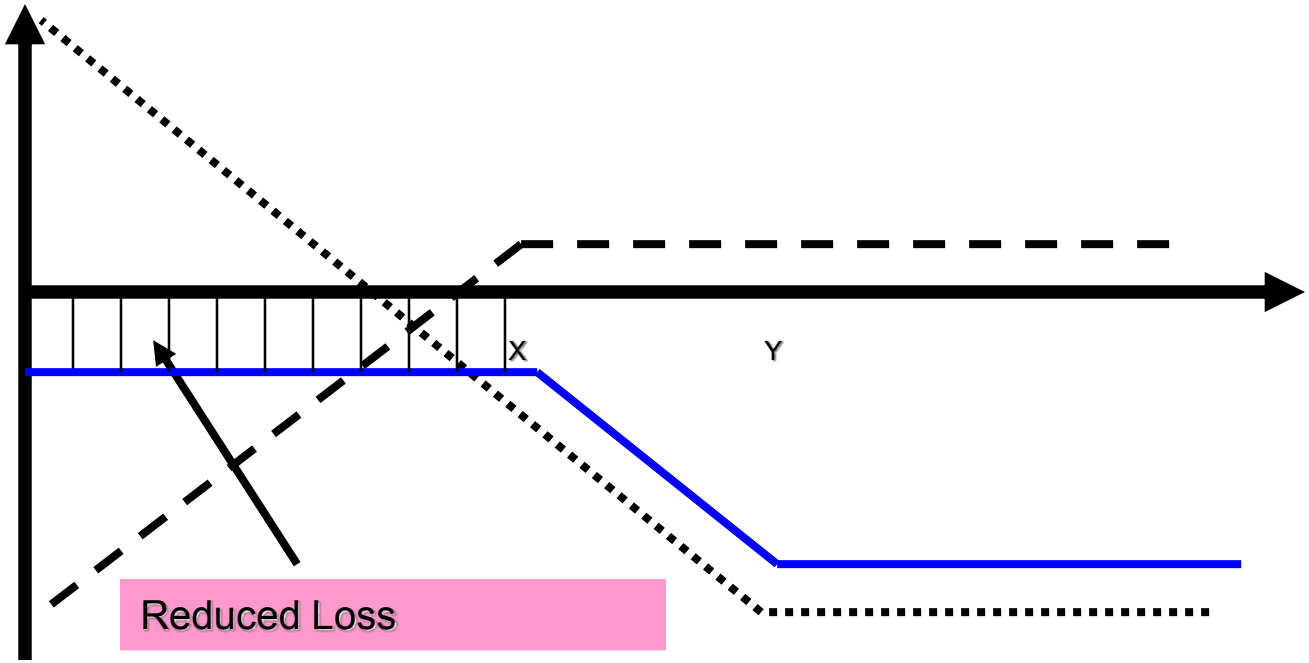
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Combinations

- Calendar Spreads
- Diagonal Spread
- Straddle
- Strips
- Straps
- Strangles

Example 8.1

A call option with a strike price of $50 costs $2. A put option with a strike price of $45 costs $3. Explain how a strangle can be created from these two options. What is the pattern of profit from the strangle?

Example 8.2

Suppose that put options on a stock with strike prices $30 and $35 cost $4 and $7 respectively. How can the options be used to create (i) a bull spread, (ii) a bear spread? Construct a table that shows the profit and payoff from both spreads.

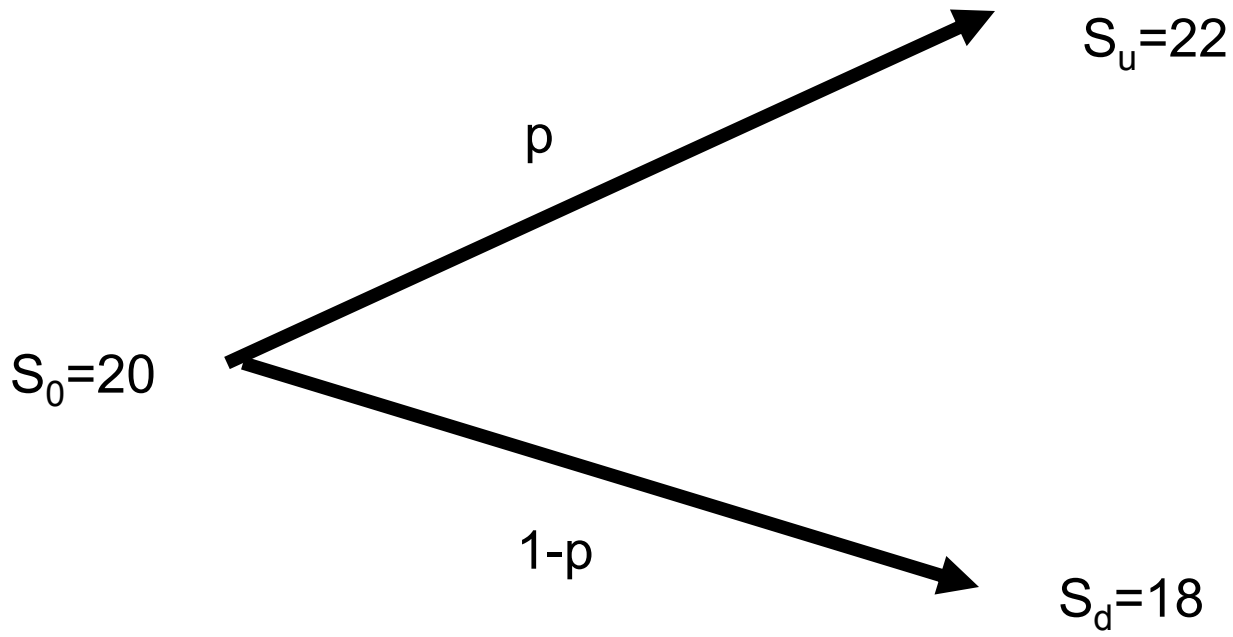
Chapter 9: Introduction to Binomial Trees

We will cover and discuss about

- One step binomial model
- Risk neutral valuation
- Two step binomial trees

One step Binomial tree

Consider the stock prices can either move up or down, one step



One step Binomial tree

Let us consider the example with the tree diagram illustrated in the previous slide. Let the portfolio consist of long position in Δ shares of stock and a short position in one call option. We will find the value of Δ which would make the portfolio riskless.

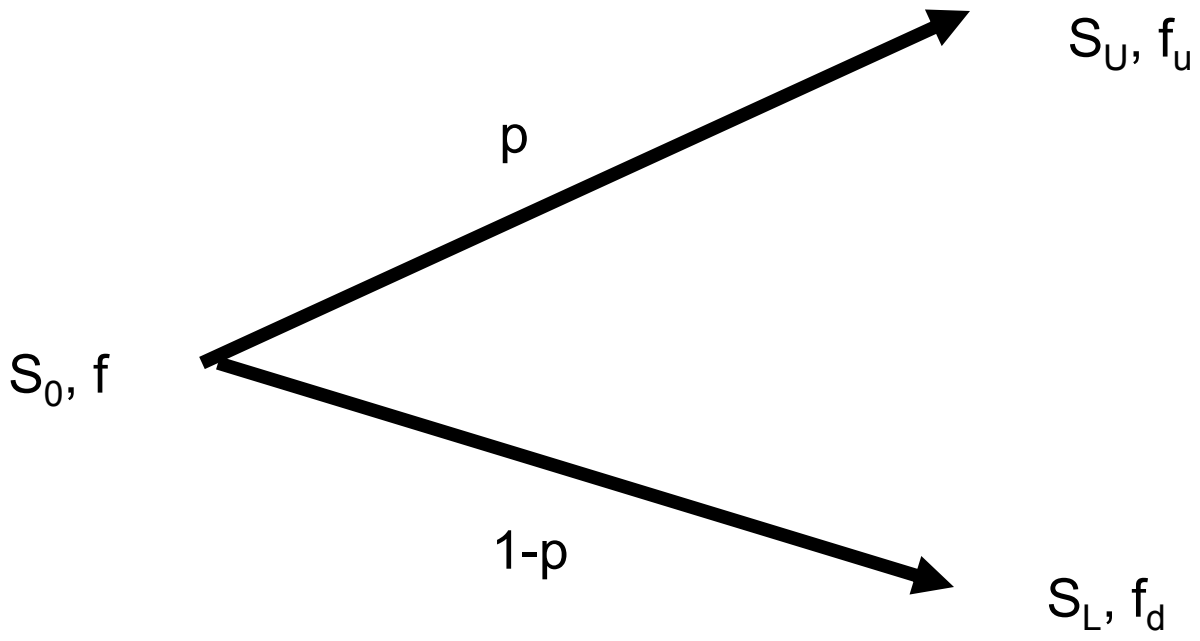
For upward price movement the value of portfolio is $22*\Delta-1$

For downward price movement the value of portfolio is $18*\Delta$

Thus $22*\Delta-1= 18*\Delta$, i.e., $\Delta=0.25$

One step Binomial tree

Consider the stock prices can either move up or down, one step



One step Binomial tree

Let the portfolio consist of long position in Δ shares (price S_0) of stock and a short position in one option (price f). We find the value of Δ which would make the portfolio riskless.

- For upward price movement (u>1), (i) the price of stock is S_0*u and (ii) payoff from option is f_u. Hence the value of portfolio is $S_0*u*\Delta-f_u$
- For downward price movement (d<1), (i) the price of stock is S_0*d and (ii) payoff from option is f_d. Hence the value of portfolio is $S_0*d*\Delta-f_d$

One step Binomial tree

- For no arbitrage we have $S_0*u*\Delta - f_u = S_0*d*\Delta - f_d$, hence $\Delta = (f_u - f_d)/(S_0*u - S_0*d)$
- Also if risk free interest rate is r, then we must have $S_0*\Delta - f = (S_0*u*\Delta - f_u)e^{-rT}$
- Hence $f = e^{-rT}[p*f_u + (1-p)*f_d]$, where $p=(e^{rT}-d)/(u-d)$

Risk Neutral valuation

$$E(S_T) = S_0 * e^{rT}$$

Example 9.1

A stock price is currently $40. It is known that at the end of one month it will be either $42 or $38. The risk free interest rate is 8% per annum with continuous compounding. What is the value of a one month European call option with a strike price of $39?

Example 9.2

Explain the no-arbitrage and risk neutral valuation approaches to valuing a European option using one step binomial tree.